HW #2

$\boxed{7.9/364}$

ⓐ $\sigma = 1$, $n = 16$

$P(|\bar{Y}-\mu| \le 0.3) = P(-0.3 \le \bar{Y}-\mu \le 0.3) = P\left(-\frac{0.3}{\sigma/\sqrt{n}} \le \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \le \frac{0.3}{\sigma/\sqrt{n}}\right) = P\left(-\frac{0.3}{1/\sqrt{16}} \le Z \le \frac{0.3}{1/\sqrt{16}}\right) = P(-1.2 \le Z \le 1.2) =$

$= 1 - 2 \times P(Z > 1.2) = 1 - 2 \times 0.1151 = 0.7698$

ⓑ $n = 25$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{\sigma/\sqrt{n}} \le Z \le \frac{0.3}{\sigma/\sqrt{n}}\right) = P\left(-\frac{0.3}{1/\sqrt{25}} \le Z \le \frac{0.3}{1/\sqrt{25}}\right) =$

$= P(-1.5 \le Z \le 1.5) = 1 - 2 \times P(Z > 1.5) = 1 - 2 \times 0.0668 = 0.8664$

$n = 36$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{1/\sqrt{36}} \le Z \le \frac{0.3}{1/\sqrt{36}}\right) = P(-1.8 \le Z \le 1.8) =$

$= 1 - 2 \times 0.0359 = 0.9282$

$n = 49$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{1/\sqrt{49}} \le Z \le \frac{0.3}{1/\sqrt{49}}\right) = P(-2.1 \le Z \le 2.1) =$

$= 1 - 2 \times 0.0179 = 0.9642$

$n = 64$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{1/\sqrt{64}} \le Z \le \frac{0.3}{1/\sqrt{64}}\right) = P(-2.4 \le Z \le 2.4) =$

$= 1 - 2 \times 0.0082 = 0.9836$

ⓒ The probabilities increase with n

ⓓ Yes. In Ex 7.3 $\begin{cases} P(|\bar{Y}-\mu| \le 0.3) \simeq 0.85 \text{ and} \\ n \simeq 43 \end{cases}$

7.10/364

(a) $\sigma = 2$, $n = 9$

$P(|\bar{Y}-\mu| \le 0.3) = P(-0.3 \le \bar{Y}-\mu \le 0.3) = P\left(\frac{-0.3}{\sigma/\sqrt{n}} \le \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \le \frac{0.3}{\sigma/\sqrt{n}}\right) =$

$= P\left(-\frac{0.3}{2/\sqrt{9}} \le Z \le \frac{0.3}{2/\sqrt{9}}\right) = P(-0.45 \le Z \le 0.45) = 1 - 2 \times P(Z > 0.45) =$

$= 1 - 2 \times 0.3264 = 0.3472$ (smaller value)

(b) $n = 25$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{2/\sqrt{25}} \le Z \le \frac{0.3}{2/\sqrt{25}}\right) = P(-0.75 \le Z \le 0.75) =$

$= 1 - 2 \times P(Z > 0.75) = 1 - 2 \times 0.2266 = 0.5468$

$n = 36$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{2/\sqrt{36}} \le Z \le \frac{0.3}{2/\sqrt{36}}\right) = P(-0.9 \le Z \le 0.9) =$

$= 1 - 2 \times P(Z > 0.9) = 1 - 2 \times 0.1841 = 0.6318$

$n = 49$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{2/\sqrt{49}} \le Z \le \frac{0.3}{2/\sqrt{49}}\right) = P(-1.05 \le Z \le 1.05) =$

$= 1 - 2 \times P(Z > 1.05) = 1 - 2 \times 0.1469 = 0.7062$

$n = 64$

$P(|\bar{Y}-\mu| \le 0.3) = P\left(-\frac{0.3}{2/\sqrt{64}} \le Z \le \frac{0.3}{2/\sqrt{64}}\right) = P(-1.2 \le Z \le 1.2) =$

$= 1 - 2 \times P(Z > 1.2) = 1 - 2 \times 0.1151 = 0.7698$

(c) The probabilities increase with n (i.e. $\uparrow n \Rightarrow \uparrow P$)

(d) Smaller probabilities with larger standard deviation (i.e. $\uparrow \sigma \Rightarrow \downarrow P$)

$\boxed{7.11/364}$ $\sigma = 4$, $n = 9$

$$P(|\bar{Y}-\mu| \le 2) = P(-2 \le \bar{Y}-\mu \le 2) = P\left(-\frac{2}{\sigma/\sqrt{n}} \le \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \le \frac{2}{\sigma/\sqrt{n}}\right) =$$

$$= P\left(-\frac{2}{4/\sqrt{9}} \le Z \le \frac{2}{4/\sqrt{9}}\right) = P(-1.5 \le Z \le 1.5) = 1-2\times P(Z>1.5) =$$

$$= 1-2\times 0.0668 = 0.8664$$

$\boxed{7.12/365}$

$$0.9 = P(|\bar{Y}-\mu| \le 1) = P(-1 \le \bar{Y}-\mu \le 1) = P\left(-\frac{1}{\sigma/\sqrt{n}} \le \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \le \frac{1}{\sigma/\sqrt{n}}\right) =$$

$$= P\left(-\frac{\sqrt{n}}{4} \le Z \le \frac{\sqrt{n}}{4}\right) = 1-2\times P\left(Z > \frac{\sqrt{n}}{4}\right) = 0.9$$

$$\Rightarrow P\left(Z > \frac{\sqrt{n}}{4}\right) = \frac{1-0.9}{2} = 0.05 \Rightarrow \frac{\sqrt{n}}{4} = 1.645 \Rightarrow \sqrt{n} = 4\times 1.645$$

$$\Rightarrow n = 43.2964 \Rightarrow n = 44$$

$\boxed{7.13/365}$ $\sigma^2 = 0.4$, $n = 10$

$$P(|\bar{Y}-\mu| \le 0.5) = P(-0.5 \le \bar{Y}-\mu \le 0.5) = P\left(-\frac{0.5}{\sigma/\sqrt{n}} \le \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \le \frac{0.5}{\sigma/\sqrt{n}}\right) =$$

$$= P\left(-\frac{0.5}{\sqrt{0.4}/\sqrt{10}} \le Z \le \frac{0.5}{\sqrt{0.4}/\sqrt{10}}\right) = P(-2.5 \le Z \le 2.5) = 1-2\times P(Z>2.5) =$$

$$= 1-2\times 0.0062 = 0.9876$$

$\boxed{7.14/365}$

$$0.95 = P(|\bar{Y}-\mu| \le 0.5) = P(-0.5 \le \bar{Y}-\mu \le 0.5) = P\left(-\frac{0.5}{\sigma/\sqrt{n}} \le \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \le \frac{0.5}{\sigma/\sqrt{n}}\right)$$

$$= P\left(-\frac{0.5\sqrt{n}}{\sqrt{0.4}} \le Z \le \frac{0.5\sqrt{n}}{\sqrt{0.4}}\right) = 1-2\times P\left(Z > \frac{0.5\sqrt{n}}{\sqrt{0.4}}\right) = 0.95$$

$$\Rightarrow P\left(Z > \frac{0.5\sqrt{n}}{\sqrt{0.4}}\right) = \frac{1-0.95}{2} = 0.025$$

$$\Rightarrow \frac{0.5\sqrt{n}}{\sqrt{0.4}} = 1.96 \Rightarrow \sqrt{n} = 2.479225 \Rightarrow n = 6.14656 \Rightarrow n = 7$$

7.15/365

$X_i \sim$ Normal (μ_1, σ_1^2)

$Y_i \sim$ Normal (μ_2, σ_2^2)

We are using Theorem 6.3/p.321 and Theorem 7.1/p.353

(a) $E(\bar{X}-\bar{Y}) \overset{Th.6.3}{=\!=} E(\bar{X}) - E(\bar{Y}) \overset{Th.7.1}{=\!=} \mu_1 - \mu_2$

(b) $V(\bar{X}-\bar{Y}) \overset{Th.6.3}{=\!=} V(\bar{X}) + V(\bar{Y}) \overset{Th.7.1}{=\!=} \frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}$

i.e $\bar{X}-\bar{Y} \sim$ Normal $\left(\mu_1-\mu_2, \frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}\right)$

(c) Since $\sigma_1^2 = 2$, $\sigma_2^2 = 2.5$, $m = n$

$\Rightarrow \sigma^2_{\bar{X}-\bar{Y}} = \frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n} = \frac{2}{n} + \frac{2.5}{n} = \frac{4.5}{n} \Rightarrow \sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{4.5}{n}}$

$0.95 = P\left(|(\bar{X}-\bar{Y})-(\mu_1-\mu_2)| \le 1\right) = P\left(-1 \le (\bar{X}-\bar{Y}) - (\mu_1-\mu_2) \le 1\right) =$

$= P\left(-\frac{1}{\sigma_{\bar{X}-\bar{Y}}} \le \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sigma_{\bar{X}-\bar{Y}}} \le \frac{1}{\sigma_{\bar{X}-\bar{Y}}}\right) =$

$= P\left(-\sqrt{\frac{n}{4.5}} \le Z \le \sqrt{\frac{n}{4.5}}\right) = 1 - 2 \times P\left(Z > \sqrt{\frac{n}{4.5}}\right) = 0.95$

$\Rightarrow P\left(Z > \sqrt{\frac{n}{4.5}}\right) = \frac{1-0.95}{2} = 0.025 \Rightarrow \sqrt{\frac{n}{4.5}} = 1.96 \Rightarrow n = 17.2872$

$\Rightarrow$ the two samples sizes should be at least 18.