STA 4322/5328

Spring 2011

Sample Exam

Full Name: _____

On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: _____

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one letter-sized sheet (the same size as the lecture notes) of your own notes and a pocket calculator. You may *not* use any books, other references, or text-capable electronic devices.

1. Suppose $Y_1, Y_2, \cdots, Y_{100}$ is a random sample from a population which is normal with mean μ and variance σ^2. Suppose $\sigma^2 = 1600$.

   (a) Provide the standard unbiased estimate of μ. [2 pts]

   (b) Provide the mean squared error of this estimate. You need to provide an explicit number. [3 pts]

   (c) Provide $P(|\bar{Y} - \mu| < 0.49)$. You need to provide an explicit number. [5 pts]

2. Suppose $Y_1, Y_2, \cdots, Y_{100}$ is a random sample from a population which is normal with mean μ and variance σ^2. Note that the standard unbiased estimate for σ^2 is given by the adjusted sample variance S^2.

   (a) Provide the mean squared error of S^2. [2 pts]

   (b) What is the distribution of $\frac{(n-1)S^2}{\sigma^2}$? [3 pts]

   (c) Provide $E[S^4]$.
   Hint: Use the fact that if X has a Gamma distribution with parameters α and β, then $E[X] = \alpha(\alpha+1)\beta^2$. [5 pts]

3. Assume that $Y_1, Y_2, \cdots, Y_{100}$ is a random sample from an exponential distribution with mean θ. It can be established that $\frac{2}{\theta}\sum_{i=1}^{n} Y_i$ is a pivotal quantity for θ, and has a χ^2-distribution with 200 degrees of freedom.

   (a) Use the pivotal quantity $\frac{2}{\theta}\sum_{i=1}^{n} Y_i$ to derive a 95% confidence interval for θ. Use the fact that if W has a χ^2-distribution with 200 degree of freedom, then $P(1 \leq W \leq 235) = 0.95$. [7 pts]

   (b) If a particular sample has $\bar{Y} = 80$, provide a confidence interval for θ. You need to provide explicit numbers. [3 pts]

4. Assume that $Y_1, Y_2, \cdots, Y_{100}$ is a sample of size n from a gamma-distributed population with parameters $\alpha = 2$ and unknown β. It can be established that $\frac{2}{\beta}\sum_{i=1}^{n} Y_i$ is a pivotal quantity for β, and has a χ^2-distribution with 400 degrees of freedom.

(a) Use the pivotal quantity $\frac{2}{\beta}\sum_{i=1}^{n} Y_i$ to derive a 95% confidence interval for β. Use the fact that if W has a χ^2-distribution with 400 degree of freedom, then $P(1 \leq W \leq 448) = 0.95$. [7 pts]

(b) If a particular sample has $\bar{Y} = 200$, provide a confidence interval for β. You need to provide explicit numbers. [3 pts]