STA 4322/5328

Spring 2011

Sample Exam

Full Name: ____

On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: ______________________________

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one letter-sized sheet (the same size as the lecture notes) of your own notes and a pocket calculator. You may *not* use any books, other references, or text-capable electronic devices.

1. Suppose $Y_1, Y_2, \cdots, Y_{100}$ is a random sample from a population with mean μ and variance σ^2. Suppose $\sigma^2 = 1600$.

   (a) Provide an approximate large sample confidence interval for μ with confidence level 95%. [7 pts]

   (b) If a particular sample has $\bar{Y} = 30$, provide an approximate large sample confidence interval with confidence level 95%. You need to provide an explicit number. [3 pts]

2. Suppose Y_1, Y_2, Y_3 is a random sample of size 3 from a population which is exponential with mean θ. Consider the estimators

$$\hat{\theta}_1 = \frac{Y_1 + Y_2}{2}, \text{ and } \hat{\theta}_2 = \bar{Y}.$$

   (a) Show that $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of θ. [5 pts]

   (b) Provide the relative efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$. [5 pts]

3. Assume that $Y_1, Y_2, \cdots, Y_n$ is a random sample from the probability density function

$$f(y) = \begin{cases} \theta y^{\theta-1} & \text{if } 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show that $E[Y_1] = \frac{\theta}{\theta+1}$. [5 pts]

   (b) Show that $\bar{Y}$ is a consistent estimator of $\frac{\theta}{\theta+1}$. [5 pts]

4. Assume that $Y_1, Y_2, \cdots, Y_{100}$ is a random sample of size n from a population which is Uniform on the interval $(0, \theta)$.

   (a) Show that $Y_{(n)} = max(Y_1, Y_2, \cdots, Y_n)$ is a sufficient estimator for θ. [7 pts]

   (b) Provide the MVUE estimator for θ. You can use the fact that $E[Y_{(n)}] = \frac{n}{n+1}\theta$. [3 pts]