STA 4322

Spring 2015

Sample Exam

Full Name: ______________________________

On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: ______________________________

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one A4-sized sheet (both sides) of your own notes and a pocket calculator. You may *not* use any books, other references, or text-capable electronic devices.

1. Suppose that the average life of an electronic component is believed to be 10 years. A researcher claims that due to recent improvements in the production process, the average life is now more than 10 years. To verify this claim, a random sample of 100 components is taken from the population. The observed sample mean is 16 years, and the observed sample standard deviation is 7.

   (a) Formulate the above problem as a statistical hypothesis testing problem. [2 points]

   (b) What is an appropriate statistic to use the standard test for the above testing problem? [3 points]

   (c) Using the standard level-0.05 test for this problem, is there enough evidence to accept the researcher's claim? Use the fact that $z_{0.95} = 1.68$. [5 points]

2. Suppose $Y_1, Y_2, \cdots, Y_{36}$ is a random sample from a population with mean μ. We want to test $H_0 : \mu = 18$ vs. $H_A : \mu > 18$ with level 0.05. Suppose the observed sample standard deviation is 5.

   (a) With the current sample, evaluate the Type II error probability for the standard level-0.05 test at $\mu_A = 20$. You can leave your answer in terms of the Φ function. Use $z_{0.95} = 1.68$. [5 points]

   (b) What is the minimum sample size required to obtain $\beta(\mu_A) = 0.01$? Use $z_{0.99} = 2.3$. [5 points]

3. Suppose we are interested in testing $H_0 : \theta = 10$ vs. $H_A : \theta > 10$, for data from some population with unknown parameter θ. It is known that there is an estimator $\hat{\theta}$ such that $\frac{\hat{\theta}-\theta}{\hat{SE}(\hat{\theta})}$ has an approximate Normal$(0, 1)$ distribution.

   (a) For what values (large or small) of $\hat{\theta}$ will we reject the null hypothesis? [2 points]

   (b) If the observed value of $\hat{\theta}$ is 11 and $\hat{SE}(\hat{\theta})$ is 5, what is the p-value of the standard test? You can leave your answer in terms of the Φ function. [5 points]

   (c) Based on this p-value, will you reject H_0 at level-0.01? You may use the fact that $1 - \Phi(0.2) > 0.01$. [3 points]

4. Suppose $Y_1, Y_2, \cdots, Y_n$ is a random sample from the Exponential(θ) density. Consider testing $H_0 : \theta = \theta_0$ vs. $H_A : \theta = \theta_A$, where $\theta_A < \theta_0$.

   (a) Use the Neymann-Pearson lemma to show that the most powerful level-α test rejects for small values of $\sum_{i=1}^{n} Y_i$. You do not need to solve for the cutoff point of the rejection region. [7 points]

   (b) Does the rejection region depend on θ_A? [3 points]