STA 4322

Spring 2020

Sample Exam

Full Name: ______________________________

On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: ______________________________

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one A4-sized sheet (both sides) of your own notes and a pocket calculator. You may *not* use any books, other references, or text-capable electronic devices.

1. Let $X_1, X_2, \cdots, X_m$ denote a random sample from the exponential density with mean θ_1, and $Y_1, Y_2, \cdots, Y_n$ denote an independent random sample from an exponential density with mean θ_2. Find the likelihood ratio criterion for testing $H_0 : \theta_1 = \theta_2$ vs. $H_A : \theta_1 \neq \theta_2$. [10 points]

2. The median sale prices for new single-family houses from 1972 to 1979 are given by $27.6, 32.5, 35.9, 39.3, 44.2, 48.8, 55.7, 62.9$ (in thousands) respectively. Let Y_i denote the median sales price, and $X_i = i$ for year i, as i varies from 1972 to 1979. Suppose we fit the linear model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

for $i = 1972, \cdots, 1979$, where the errors $\epsilon_1, \epsilon_2, \cdots, \epsilon_8$ are assumed to be independent and identically distributed with mean 0 and variance σ^2.

(a) Find the least squares estimators of β_0 and β_1. [5 points]

(b) Find the estimate of the error variance σ^2. [5 points]

3. Consider the same setup as Problem 2 above, and assume that the errors are normally distributed.

(a) Find the 90% confidence interval for β_0. Use $t_{6,0.95} = 1.72$. [5 points]

(b) Find the 95% confidence interval for $\beta_0 + \beta_1$. Use $t_{6,0.975} = 2$ [5 points]

4. Suppose we have data $(X_1, Y_1), (X_2, Y_2), \cdots, (X_{100}, Y_{100})$, and we fit a linear model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

for $i = 1, 2, \cdots, 100$. Suppose $\bar{X} = 10, \bar{Y} = 30$, $\sum_{i=1}^{100}(X_i - \bar{X})(Y_i - \bar{Y}) = 50$, $\sum_{i=1}^{100}(X_i - \bar{X})^2 = 70$, and $\sum_{i=1}^{100}(Y_i - \bar{Y})^2 = 40$.

(a) Find the values of the least squares estimators $\hat{\beta}_0$ and $\hat{\beta}_1$. [3 points]

(b) Find the 95% confidence interval for $\beta_0 + 35\beta_1$. Use $t_{98,0.975} = 1.96$. [7 points]