Solutions to Quiz 1

1. If $Y_1, Y_2, \cdots, Y_n$ is a random sample from a population with mean μ and variance σ^2, and $\bar{Y}$ denotes the sample mean, then the MSE of $\bar{Y}$ is given by

   (a) σ^2.

   (b) n.

   (c) $\frac{\sigma^2}{n}$.

   (d) $n\sigma^2$.

   Answer: (c): See lecture notes.

2. The sample mean estimator $\bar{Y}$ (in Problem 1 above) is unbiased. This statement is

   (a) True.

   (b) False.

   Answer: (a): See lecture notes.

3. In Problem 1 above, $E[\bar{Y}^2]$ is given by

   (a) μ^2.

   (b) $\mu^2 + \frac{\sigma^2}{n}$.

   (c) σ^2.

   (d) $\frac{\sigma^2}{n}$.

   Answer: (b):

$$E[\bar{Y}^2] = V(\bar{Y}) + (E[\bar{Y}])^2 = \frac{\sigma^2}{n} + \mu^2.$$

4. In Problem 1 above, let

$$\hat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2.$$

   Then $E[\hat{\sigma^2}]$ is given by

   (a) σ^2.

   (b) $n\sigma^2$.

   (c) $\frac{n}{n-1}\sigma^2$.

   (d) $\frac{n-1}{n}\sigma^2$.

   Answer: (d): See lecture notes.

5. In Problem 1 above, the probability distribution of $\bar{Y}$ is always Poisson. This statement is

   (a) True.
   (b) False.

   Answer: (b): No assumption about the distribution of $Y_1, Y_2, \cdots, Y_n$ has been made in Problem 1.