SOLUTIONS TO EXAM-2

Problem 1: (a) X is a Binomial random variable with parameters $n = 4196$ and $p = 0.5$ (fair coin).

(b) $$E(X^2) = V(X) + (E(X))^2$$
$$= np(1-p) + (np)^2$$
$$= \frac{4196}{4} + \left(\frac{4196}{2}\right)^2$$
$$= 4402653.$$

(c) If the number of heads is equal to the number of tails, then $X = \frac{4196}{2} = 2098$.

Hence,
$$P(\#\text{ of heads} = \#\text{ of tails})$$
$$= P(X = 2098)$$
$$= \binom{4196}{2198} (0.5)^{2098} (1-0.5)^{4196-2098}$$
$$= (4196!/2098!\,2098!)\,(0.5)^{4196}.$$

Problem 2: (a) X is a geometric random variable with parameter $p = 0.2$ (or alternatively a negative binomial random variable with parameters $r = 1$ and $p = 0.2$).

(b) P(Atleast 9 more failed sales | 5 failed sales)

$= P(X \geq 9+5 \mid X \geq 5)$

$= P(X \geq 9)$ ($\because$ By the Memoryless Property)

$= (1-p)^9$

$= (0.8)^9$.

Alternatively, we can use a direct computation...

$$P(X \geq 9+5 \mid X \geq 5) = P(X \geq 14 \mid X \geq 5)$$

$$= \frac{P(\{X \geq 14\} \cap \{X \geq 5\})}{P(X \geq 5)}$$

$$= \frac{P(X \geq 14)}{P(X \geq 5)}$$

$$= \frac{(0.8)^{14}}{(0.8)^5}$$

Hence,

$$P(X \geq 9+5 \mid X \geq 5) = (0.8)^9.$$

(c) Total number of customers the saleswoman has to approach $= X+1$.

$$E(X+1) = E(X)+1$$
$$= \frac{1-p}{p} + 1$$
$$= 5.$$

$$V(X+1) = V(X)$$
$$= \frac{1-p}{p^2}$$
$$= 20.$$

Problem 3: (a) $P(X=25) = \dfrac{e^{-49}(49)^{25}}{25!}$.

(b) $P(X \leq 1) = P(X=0) + P(X=1)$

$$= \frac{e^{-49}(49)^0}{0!} + \frac{e^{-49}(49)^1}{1!}$$

$$= 50e^{-49}.$$

(c) $$P(X=0 \mid X \leq 1) = \frac{P(\{X=0\} \cap \{X \leq 1\})}{P(X \leq 1)}$$

$$= \frac{P(X=0)}{P(X \leq 1)}$$

$$= \frac{\dfrac{e^{-49}(49)^0}{0!}}{50e^{-49}}$$

$$= \frac{1}{50}$$

$$= 0.02.$$

<u>Problem 4:</u> (a) X is a hypergeometric random variable with parameters $N = 30$, $k = 4$, $n = 18$.

(b) P(All 18 PC's are not defective)

$$= P(X=0)$$

$$= \frac{\binom{k}{0}\binom{N-k}{n-0}}{\binom{N}{n}}$$

$$= \frac{\binom{4}{0}\binom{26}{18}}{\binom{30}{18}}$$

$$= \frac{\binom{26}{18}}{\binom{30}{18}}.$$

(c) $$E[(X+4)^2] - V(X) = E[X^2 + 8X + 16] - V(X)$$

$$= E[X^2] + 8E[X] + 16 - V(X)$$

$$= V(X) + (E[X])^2 + 8E[X] + 16 - V(X)$$

$$= (E[X])^2 + 8E[X] + 16$$

$$= \left(\frac{nk}{N}\right)^2 + 8\left(\frac{nk}{N}\right) + 16$$

$$= \left(\frac{12}{5}\right)^2 + 8\left(\frac{12}{5}\right) + 16$$

$$= \frac{144}{25} + \frac{96}{5} + 16$$

$$= 40.96$$