Solutions to Homework 1

2.10

a. The two jobs are identical, so the order does not matter when selecting two applicants, the sample space is

$$S=\{(\text{Jim, Don}),\ (\text{Jim, Mary}),\ (\text{Jim, Sue}),\ (\text{Jim, Nancy}),\ (\text{Don, Mary}),\ (\text{Don, Sue}),\ (\text{Don, Nancy}),\ (\text{Mary, Sue}),\ (\text{Mary, Nancy}),\ (\text{Sue, Nancy})\}$$

b. Jim and/or Don should be selected, then

$$A=\{(\text{Jim, Don}),\ (\text{Jim, Mary}),\ (\text{Jim, Sue}),\ (\text{Jim, Nancy}),\ (\text{Don, Mary}),\ (\text{Don, Sue}),\ (\text{Don, Nancy})\}$$

thus there are 7 outcomes in A.

c. exactly one in Jim and Don is selected, then

$$B=\{(\text{Jim, Mary}),\ (\text{Jim, Sue}),\ (\text{Jim, Nancy}),\ (\text{Don, Mary}),\ (\text{Don, Sue}),\ (\text{Don, Nancy})\}$$

thus there are 6 outcomes in B.

d. $A\overline{B}$

e. $\overline{A}=\{(\text{Mary, Sue}),\ (\text{Mary, Nancy}),\ (\text{Sue, Nancy})\}$

$$AB=B=\{(\text{Jim, Mary}),\ (\text{Jim, Sue}),\ (\text{Jim, Nancy}),\ (\text{Don, Mary}),\ (\text{Don, Sue}),\ (\text{Don, Nancy})\}$$

$$A\cup B=A=\{(\text{Jim, Don}),\ (\text{Jim, Mary}),\ (\text{Jim, Sue}),\ (\text{Jim, Nancy}),\ (\text{Don, Mary}),\ (\text{Don, Sue}),\ (\text{Don, Nancy})\}$$

$$\overline{AB}=\{(\text{Jim, Don}),\ (\text{Mary, Sue}),\ (\text{Mary, Nancy}),\ (\text{Sue, Nancy})\}$$

2.17

Let events

E1: black molly and guppy are chosen.

E2: black molly and goldfish are chosen

E3: black molly and neon are chosen

E4: guppy and gold fish are chosen

E5: guppy and neon are chosen

E6: goldfish and neon are chosen

Then the sample space $S=E1\cup E2\cup E3\cup E4\cup E5\cup E6$. Since the girl is equally likely to choose each fish species, then

$$P(E1)=P(E2)=P(E3)=P(E4)=P(E5)=P(E6)=\frac{1}{6}$$

a. Let A denote the event that she does not choose a guppy, then $A = E2 \cup E3 \cup E6$, by axiom 3 of definition 2.3,

$$P(A) = P(E2 \cup E3 \cup E6) = P(E2) + P(E3) + P(E6) = \frac{1}{2}$$

b. $P(E5) = \frac{1}{6}$

c. Let B denote the event that she has either a black molly or a neon but not both, then

$$P(B) = P(E1 \cup E2 \cup E5 \cup E6) = P(E1) + P(E2) + P(E5) + P(E6) = \frac{2}{3}$$

2.18

Because all applicants are equally qualified, then all outcomes in the sample space should be assigned to the same probability, $\frac{1}{10}$.

a. the event that both males are selected has only one outcome, (Jim, Don), thus

$$P(\text{both males are selected}) = 1 \cdot \frac{1}{10} = 0.1$$

b. A has 7 outcomes, thus

$$P(\text{at lease one male is selected}) = P(A) = 7 \cdot \frac{1}{10} = 0.7$$

c. the event at lease one female is selected has all outcomes in S except (Jim, Don)

$$P(\text{at least one female is selected}) = 9 \cdot \frac{1}{10} = 0.9$$

2.20.

30% of all potential customers buy products from outlet alone, thus, for a given potential customer,

$$\begin{aligned} 0.3 &= P(\text{the customer buys from outlet 1 only}) \\ &= P(\text{the customer buys from outlet 1 and the customer does not buy from outlet 2}) \\ &= P(A\bar{B}) \end{aligned}$$

40% buy from outlet 2 alone, then we have $P(\bar{A}B) = 0.4$

10% of potential customers buy from both 1 and 2, then $P(AB) = 0.1$

a.

$$\begin{aligned} P(\text{the customer buys from outlet 1}) &= P(A) \\ &= P(AB) + P(A\bar{B}) \\ &= 0.1 + 0.3 \\ &= 0.4 \end{aligned}$$

b.

$$
\begin{aligned}
P(\text{the customer does not buy from outlet 2}) &= P(\overline{B}) \\
&= 1 - P(B) \\
&= 1 - P(AB) - P(\overline{A}B) \\
&= 1 - 0.1 - 0.4 \\
&= 0.5
\end{aligned}
$$

c.

$$
\begin{aligned}
P(\text{the customer does not buy from outlet 1 or from outlet 2}) &= P(\overline{A} \cup \overline{B}) \\
&= P(\overline{A \cap B}),\ \text{we use DeMorgan's law here} \\
&= 1 - P(AB) \\
&= 1 - 0.1 \\
&= 0.9
\end{aligned}
$$

d.

$$
\begin{aligned}
P(\text{the customer does not buy from outlet 1 and from outlet 2}) &= P(\overline{A} \cap \overline{B}) \\
&= P(\overline{A \cup B}),\ \text{we use DeMorgan's law here} \\
&= 1 - P(A \cup B) \\
&= 1 - P(A) - P(\overline{A}B) \\
&= 1 - 0.4 - 0.4 \\
&= 0.2
\end{aligned}
$$

2.21

For the first person who shows up tomorrow to donate blood,

Let B denote the event that blood type is O^{+}

Let C denote the event that blood type is O

Let D denote the event that blood type is O^{-}

Let E denote the event that blood type is A^{+}

Let F denote the event that blood type is A

Let G denote the event that blood type is A^{-}

Events B,D,E,G are mutually exclusive, Then,

a. $P(B) = \frac{1}{2}$

b. $P(C) = P(B) + P(D) = \frac{1}{2} + \frac{1}{11} = \frac{13}{22}$

c. $P(F) = P(E) + P(G) = \frac{1}{4} + \frac{1}{20} = \frac{3}{10}$

d. $P(G \cup D) = P(G) + (D) = \frac{1}{20} + \frac{1}{11} = \frac{31}{220}$

e. $P(\overline{E}\cap\overline{B}) = P(\overline{E\cup B}) = 1-P(E\cup B) = 1-P(E)-P(B) = 1-\frac{1}{4}-\frac{1}{2} = \frac{1}{4}$

2.23

If one such assembly is randomly chosen, let A denote the event that the assembly has defects in shaft, B denote the event that the assembly has defects in the bushing, then we know that

$$P(A) = 0.1, P(B) = 0.08, P(AB) = 0.02$$

a.

$$\begin{aligned} P(\text{the assembly has only a bushing defect}) &= P(\overline{A}B) \\ &= P(B) - P(AB) \\ &= 0.08 - 0.02 \\ &= 0.06 \end{aligned}$$

b.

$$\begin{aligned} P(\text{the assembly has a shaft or bushing defect}) &= P(A\cup B) \\ &= P(A) + P(B) - P(AB) \\ &= 0.1 + 0.08 - 0.02 \\ &= 0.16 \end{aligned}$$

c. If the assembly has only one of two types of defects, it has only the shaft defect or has only the bushing defect, we can denote them by $A\overline{B}$ and $\overline{A}B$, they are mutually exclusive, then

$$\begin{aligned} P(\text{the assembly has only one of two types of defects}) &= P(A\overline{B}\cup\overline{A}B), \\ &= P(A\overline{B}) + P(\overline{A}B) \\ &= (P(A) - P(AB)) + (P(B) - P(AB)) \\ &= 0.08 + 0.06 \\ &= 0.14 \end{aligned}$$

d.

$$\begin{aligned} P(\text{the assembly has no defects in either shafts or bushings}) &= P(\overline{A}\cap\overline{B}) \\ &= P(\overline{A\cup B}) \\ &= 1 - P(A\cup B) \\ &= 1 - 0.16 \\ &= 0.84 \end{aligned}$$

2.35

The two customers are denoted by A and B

a. the sample space is

$$S = \{\text{(A uses door I, B uses door I), (A uses door I, B uses door II),}$$
$$\text{(A uses door II, B uses door I), (A uses door II, B uses door II)}\}$$

b. all outcomes have the same probability, $\frac{1}{4}$

$$P(\text{both customers use door I}) = P\left(\{\text{(A use door I, B use door I)}\}\right)$$
$$= \frac{1}{4}$$

c.

$$P(\text{customers use different doors}) = P\left(\{\text{(A use door I, B use door II)}, \text{(A use door II, B use door I)}\}\right)$$
$$= \frac{1}{4} \cdot 2$$
$$= 0.5$$