4.51

a. each individual has the probability of 0.5 to pass the gene to an offspring.

$$\begin{aligned} P(\text{the child has no disease gene}) &= P(\text{the first person do not pass gene to the child}) \\ &\quad \times P(\text{the second person do not pass gene to the child}) \\ &= 0.5\times 0.5 \\ &= 0.25 \end{aligned}$$

b. for each child, having no disease gene is an independent event, then the probability of interest is

$$0.25^5 = 0.000977$$

4.54

Let X denote the number of components last longer than 1000 hours, then X can be modeled to have a binomial distribution with $p = 1 - 0.15 = 0.85$

a. We have

$$P(X = 2) = \binom{4}{2}(0.85)^2(1-0.85)^2 = 0.0975$$

b. If the subsystem operates for longer than 1000 hours, any two or more of the four components last for more than 1000 hours, so we get

$$\begin{aligned} P(X \ge 2) &= \binom{4}{2}(0.85)^2(1-0.85)^2 + \binom{4}{3}(0.85)^3(1-0.85)^1 + \binom{4}{4}(0.85)^4(1-0.85)^0 \\ &= 0.988 \end{aligned}$$

4.58

a. X has a binomial distribution with $p = 0.1$

$$P(X \ge 1) = 1 - P(X = 0) = 1 - \binom{n}{0}(0.1)^0(1-0.1)^n = 1 - 0.9^n = 0.95$$

Thus $n = \dfrac{\log 0.05}{\log 0.9} = 28$

b. X has a binomial distribution with $p = 0.1$

$$P(X \ge 1) = 1 - P(X = 0) = 1 - \binom{n}{0}(0.05)^0(1-0.05)^n = 1 - 0.95^n = 0.95$$

Thus $n = \dfrac{\log 0.05}{\log 0.95} = 58$

4.60

For each case,

$$P(\text{innocent people receiving death penalty}) = 0.01 \times \sum_{x=2}^{3} \binom{3}{x} (0.05)^x (1-0.05)^{3-x}$$
$$= 0.0000725$$

Let X denote the number of innocent people receiving death penalty, X can be modeled as a binomial distribution with $n = 100, p = 0.0000725$, then

$$E(X) = np = 100 \times 0.0000725 = 0.00725$$
$$V(X) = np(1-p) = 100 \times 0.0000725 \times 0.9999275 = 0.007249$$
$$\sigma = \sqrt{V(X)} = 0.0851$$

4.68

Define

event A: the first two engines are defective

event B: at least four defective engines are tested before the first nondefective engine is found.

We need to find $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$

The first and second test are independent, then we have

$$P(A) = 0.1 \times 0.1 = 0.01$$

Event $B \subseteq A$, then

$$P(B \cap A) = P(B) = \sum_{k=4}^{\infty} (0.1)^k \cdot 0.9 = 0.0001$$

Therefore,

$$P(B \mid A) = \frac{0.0001}{0.01} = 0.01$$

4.78

Let Y be the number of people who refuse to be interviewed before she obtains five people, so

$Y = X - 5$, Y can reasonably be assumed to have the negative binomial distribution with $p = 0.6, r = 5$

a.

$$\begin{aligned} P(X \le 7) &= P(Y \le 2) \\ &= \sum_{i=0}^{2} \binom{i+4}{4} (0.6)^5 (1-0.6)^i \\ &= 0.419904 \end{aligned}$$

b.

$$E(Y) = \frac{rq}{p} = \frac{5\times 0.4}{0.6} = \frac{10}{3}, V(Y) = \frac{rq}{p^2} = \frac{5\times 0.4}{0.6^2} = \frac{50}{9}$$

Because $Y = X - 5$, so $X = Y + 5$, then

$$E(X) = E(Y) + 5 = \frac{25}{3}, \quad V(X) = V(Y) = \frac{50}{9}$$

4.79

Let X be the number of customers that the salesman has to visit to make three sales, let Y denote the number of customers who refuse to purchase a car before the salesman makes three sales, so $Y = X - 3$. Suppose that trials are independent, Y can be modeled as negative binomial distribution with $p = 0.2, r = 3$

a.

$$\begin{aligned} P(X \ge 5) &= P(Y \ge 2) \\ &= 1 - P(Y \le 1) \\ &= 1 - \sum_{i=0}^{1} \binom{i+2}{2} (0.2)^3 (1-0.2)^i \\ &= 0.9728 \end{aligned}$$

b.

$$E(Y) = \frac{rq}{p} = \frac{3\times 0.8}{0.2} = 12$$

Because $Y = X - 3$, so $X = Y + 3$, then

$E(X) = E(Y) + 3 = 15$

4.83

a.

Let Y_i denote the event that John wins in the i th game, $P(Y_i = 1) = p = 0.6$. Because the outcome of each game is independent of the outcomes of the other games, then

$P(i=3) = P(Y_1 = 1)P(Y_2 = 1)P(Y_3 = 1) = 0.6^3 = 0.216$

$P(i=4) = P(Y_1 = 0)P(Y_2 = 1)P(Y_3 = 1)P(Y_4 = 1) + P(Y_1 = 1)P(Y_2 = 0)P(Y_3 = 1)P(Y_4 = 1)$
$+P(Y_1 = 1)P(Y_2 = 1)P(Y_3 = 0)P(Y_4 = 1)$
$= \binom{3}{1} p^3 (1-p)^1$
$= 0.2592$

There are $\binom{4}{2}$ ways to choose two games which John lost in the first four games,then

$P(i=5) = \binom{4}{2} p^3 (1-p)^2 = 0.20736$

b.

$$P(\text{john wins in three-out-of-five series}) = P(i=3) + P(i=4) + P(i=5) = 0.68252$$

$$P(\text{john wins in two-out-of-three series}) = P(\text{john wins}|\text{three games}) + P(\text{john wins}|\text{two games}) = \binom{2}{1}(0.6)^2(1-0.6) + (0.6)^2 = 0.648$$

The probability that John wins in two-out-of-three series is smaller than the probability that he wins in three-out-of-five series .

4.84

let X denote the number of games he loses from the fourth game before he stops playing, Y denote the number of games the child plays, then $Y = X + 4$. The outcomes of games are independent, thus X can be modeled as geometric distribution with parameter p .

a. $E(X) = \frac{q}{p} = \frac{1-p}{p}$, thus $E(Y) = E(X) + 4 = \frac{1-p}{p} + 4$

b. Let Z denote the number of games that the child wins in the first three game, M denote the number of games the child will win. Then $M = Z+1$. Z can be modeled as binomial distribution with parameter p. We then have

$E(Z) = np = 3p, \quad E(M) = E(Z)+1 = 3p+1$

4.93.

Let X denote the number of customer arrivals for a given hour. X has a Possion distribution with $\lambda = 7$.

a. $p(7) = \frac{7^7}{7!}e^{-7} = 0.149$

b. $P(X \le 2) = \sum_{x=0}^{2} \frac{7^x}{x!}e^{-7} = 0.030$

c. $P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.007 = 0.993$

4.98

Let X denote the number of emitted particles for a given hour. X has a Possion distribution with $\lambda = 4$.

a. $P(X \ge 6) = 1 - P(X \le 5) = 1 - 0.785 = 0.215$

b. $P(X \le 3) = 0.433$

c. Let Y denote the number of emitted particles in a given 24-hour period. X has a Possion distribution with $\lambda = 96$. Then

$$P(Y = 0) = \frac{96^0}{0!}e^{-96} = e^{-96}$$

4.100

Let X denote the number of fatalities in a given year. X has a Possion distribution with $\lambda = 4.4$. Let Y denote the number of fatalities during the five-year period. Y has a Possion distribution with $\lambda = 4.4 \times 5 = 22$.

a. $P(X = 0) = \frac{4.4^0}{0!}e^{-4.4} = 0.012$

b. $P(X \ge 6) = 1 - P(X \le 5) = 1 - 0.720 = 0.280$

c. $P(Y = 0) = \frac{22^0}{0!}e^{-22} = e^{-22}$

d. $P(Y \le 12) = 0.015$

4.104

X denotes the number of breakdowns during t hours of operation. Suppose the breakdowns are independent, then X can be modeled as a Possion distribution with $\lambda = \frac{2}{10}t = 0.2t$ .We then have

$$E(X) = \lambda = 0.2t, \quad V(X) = \lambda = 0.2t$$

Thus

$$E(X^2) = V(X) + [E(X)]^2 = 0.2t + (0.2t)^2 = 0.2t + 0.04t^2$$

The expected value between shutdowns is

$$\begin{aligned} E(R) &= E(300t - 75X^2) \\ &= 300t - 75E(X^2) \\ &= 300t - 15t - 3t^2 \\ &= -3\left(t - \frac{95}{2}\right)^2 + 3\cdot\left(\frac{95}{2}\right)^2 \\ &\le 3\cdot\left(\frac{95}{2}\right)^2 \end{aligned}$$

When $t_0 = \frac{95}{2}$, we obtain the maxim expected revenue between shutdowns.

4.115

Let X denote the number of selected firms which are not local. Because three firms are randomly chosen without replacement, X has a hypergeometric distribution with $N = 6, k = 2, n = 3$

a.

$$P(X = 0) = \frac{\binom{2}{0}\binom{6-2}{3-0}}{\binom{6}{3}} = \frac{1}{5}$$

b.

$$P(X \ge 1) = 1 - P(X = 0) = \frac{4}{5}$$

4.120

Let X denote the number of selected PCs which do not function properly. Suppose that five PCs are randomly chosen, X has a hypergeometric distribution with

$N=10, k=4, n=5$. When all five are not defective, $X=0$

$$P(X=0)=\frac{\binom{4}{0}\binom{10-4}{5-0}}{\binom{10}{5}}=\frac{1}{42}$$