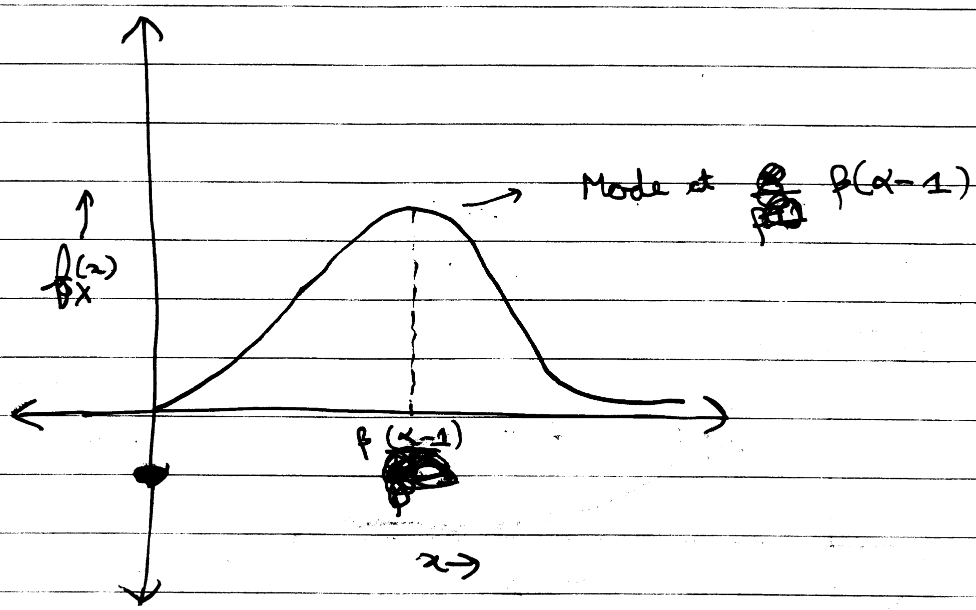
LECTURE - 23

Additional notes on the Gamma distribution

Note that if X is Gamma(α, β), then the density of X is given by

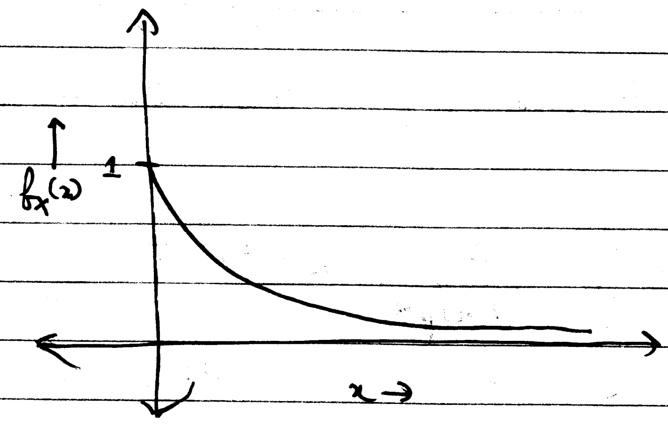
$$f_X(x) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}} & \text{if } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

We would like to understand what α and β represent. The parameter α is called the "shape parameter", and governs the shape of f_X. If $\alpha > 1$, the graph of f_X looks as follows.



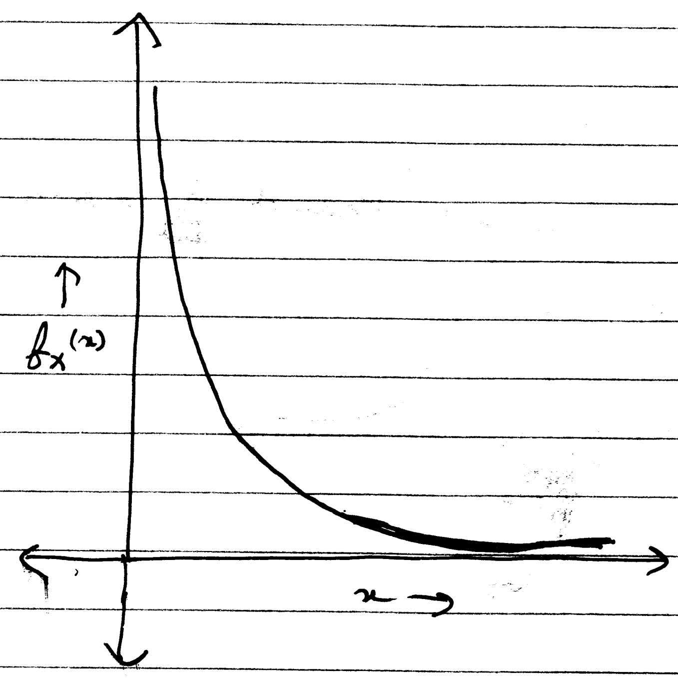
$$\boxed{\alpha > 1}$$

If $\alpha = 1$, we have the Exponential (β) density, which looks like the following.



$\boxed{\alpha = 1}$

If $\alpha < 1$, $f_X(x)$ has an exponential tail, and converges to ∞ when x converges to 0. The plot is provided below.



$\boxed{\alpha < 1}$

The parameter β is known as the "scale parameter".

Here is a natural method of choosing α and β if you have an approximate value for $E(X)$ and $V(X)$.

<u>Example:</u> Suppose X is Gamma (α, β), and $E(x) = 20$, $V(X) = 100$. Find α and β.

Note that $E(X) = \alpha\beta = 20$ and $V(X) = \alpha\beta^2 = 100$.

$$\Rightarrow \beta = \frac{\alpha\beta^2}{\alpha\beta} = 5.$$

$$\Rightarrow \alpha = \frac{20}{\beta} = 4.$$