# SOLUTIONS TO SAMPLE EXAM

**PROBLEM 1:** SEE SOLUTIONS TO HOMEWORK 9

**PROBLEM 2:** (a) Using a calculator, it can be computed that $\overline{Y} = 43.36$, $\overline{X} = 1975.5$,

$$\sum_{i=1}^{8}(X_i-\overline{X})(Y_i-\overline{Y}) = \sum_{i=1}^{8} X_i Y_i - 8\,\overline{X}\,\overline{Y}$$

$$= 685504.3 - 685300.90$$

$$= 203.40,$$

and

$$\sum_{i=1}^{8}(X_i-\overline{X})^2 = 42.$$

$$\boxed{\begin{array}{l}\text{USEFUL IDENTITY:}\\ \displaystyle\sum_{i=1}^{n}(X_i-\overline{X})(Y_i-\overline{Y}) = \sum_{i=1}^{n} X_i Y_i - n\,\overline{X}\,\overline{Y}\end{array}}$$

It follows that

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{8}(X_i-\overline{X})(Y_i-\overline{Y})}{\sum_{i=1}^{8}(X_i-\overline{X})^2}$$

$$= \frac{203.40}{42}$$

$$= 4.84$$

and

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1\overline{X} = 43.36 - 4.84 \times 1975.5$$

$$= -9518.06$$

1(b) By the identity established in class,

$$\hat{\sigma}^2 = \frac{1}{n-2}\left[\sum_{i=1}^{n}(Y_i-\bar{Y})^2 - \hat{\beta}_1^2 \sum_{i=1}^{n}(X_i-\bar{X})^2\right]$$

$$= \frac{1}{6}\left[1004.57 - (4.84)^2 \times 42\right]$$

$$= \frac{1}{6} \times 20.69$$

$$= 3.45$$

PROBLEM 3: 3(a) By the formula provided in Lecture notes, the 90% confidence interval for $\beta_0$ is given by

$$\left[\hat{\beta}_0 - t_{6,0.95}\,\hat{\sigma}\sqrt{\frac{1}{8}+\frac{\bar{X}^2}{\sum_{i=1}^{8}(X_i-\bar{X})^2}},\right.$$
$$\left.\hat{\beta}_0 + t_{6,0.95}\,\hat{\sigma}\sqrt{\frac{1}{8}+\frac{\bar{X}^2}{\sum_{i=1}^{8}(X_i-\bar{X})^2}}\right]$$

$$= \left[-9518.06 - 1.72 \times \sqrt{3.45}\sqrt{\frac{1}{8}+\frac{(1975.5)^2}{42}},\right.$$
$$\left.-9518.06 + 1.72 \times \sqrt{3.45}\sqrt{\frac{1}{8}+\frac{(1975.5)^2}{42}}\right]$$

$$= \left[-9518.06 - 973.85,\ -9518.06 + 973.85\right]$$

$$= \left[-10491.91,\ -8544.21\right]$$

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1(b) By the formula provided in Lecture notes, the 95% confidence interval for $\beta_0 + \beta_1$ is given by

$$\left[ \hat{\beta}_0 + \hat{\beta}_1 - t_{6,0.975}\, \hat{\sigma} \sqrt{\frac{1}{8} + \frac{(1975.5-1)^2}{42}},\right.$$
$$\left. \hat{\beta}_0 + \hat{\beta}_1 + t_{6,0.975}\, \hat{\sigma} \sqrt{\frac{1}{8} + \frac{(1975.5-1)^2}{42}} \right]$$

$$= [-9513.22 - 1131.807, -9513.22 + 1131.807]$$

$$= [-10645.03, -8381.413]$$

PROBLEM 4: 4(a) $\hat{\beta}_1 = \dfrac{\sum_{i=1}^{100} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{100} (X_i - \bar{X})^2}$

$$= \frac{5}{7}$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 30 - \frac{5}{7} \times 10$$

$$= \frac{160}{7} \approx 22.86$$

4(b) By the formula provided in Lecture notes, the 95% confidence interval for $\beta_0 + 35\beta_1$ is given by

$$\left[\hat{\beta}_0 + 35\hat{\beta}_1 - t_{98,0.975}\,\hat{\sigma}\sqrt{\frac{1}{100} + \frac{(35-10)^2}{70}}\right.,$$

$$\left.\hat{\beta}_0 + 35\hat{\beta}_1 + t_{98,0.975}\,\hat{\sigma}\sqrt{\frac{1}{100} + \frac{(35-10)^2}{70}}\right]$$

$$= \left[47.86 - 1.56 \times \sqrt{\frac{4.29}{98}} \times \sqrt{\frac{1}{100} + \frac{625}{70}}\right.,$$

$$\left.47.86 + 1.96 \times \sqrt{\frac{4.29}{98}} \times \sqrt{\frac{1}{100} + \frac{625}{70}}\right]$$

$$\left(\because \quad \hat{\sigma}^2 = \frac{40 - \left(\frac{5}{7}\right)^2 \times 70}{100 - 2} = \frac{4.29}{98}\right)$$

$$= [47.86 - 0.123,\ 47.86 + 0.123]$$

$$= [47.737,\ 47.983]$$