

STA 4321/5325

Spring 2010

Sample Exam: Continuous random variables

Full Name: _____

On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: _____

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one letter-sized sheet (the same size as the lecture notes) of your own notes and a pocket calculator. (You are not required to bring a calculator — you may leave your answers in a form from which the numerical answer could be immediately calculated.) You may *not* use any books, other references, or text-capable electronic devices.

1. Let X be an exponential random variable with mean 9.
 - (a) Provide the probability density function of X . [2 pts]
 - (b) Find $P(25 \leq X \leq 45 \mid X > 20)$. [4 pts]
 - (c) Provide $E[X(X + 3)]$. [4 pts]

2. In tests of stopping distance for automobiles, cars traveling 30 miles per hour before the brakes were applied tended to travel distances that appeared to be uniformly distributed between two points a and b . Find the probabilities of the following events.
 - (a) One of these automobiles, selected at random, stops closer to a than to b . [2 pts]
 - (b) One of these automobiles, selected at random, stops at a point where the distance to a is more than 9 times the distance to b . [4 pts]
 - (c) Suppose that three automobiles (which behave independently) are used in the test. Find the probability that exactly one of the three travels past the midpoint between a and b . [4 pts]

3. Let X be a gamma random variable with parameters α and β .
 - (a) Provide the probability density function of X . [2 pts]
 - (b) If $E(X) = 20$ and $V(X) = 100$, find α and β .
Hint: Use the formulas for $E(X)$ and $V(X)$ in terms of α and β , and solve for α and β . [4 pts]
 - (c) Using Tchebysheff's inequality, provide an interval such that the probability that X lies in this interval is at least 99%. [4 pts]

4. When fishing off the shores of Florida, a spotted sea trout must be between 14 and 30 inches long before it can be kept; otherwise it must be returned to the waters. In a region of the Gulf of Mexico, the lengths of the spotted sea trout that are caught, are normally distributed with a mean of 22 inches, and a standard deviation of 4 inches, i.e., $\mu = 22$ and $\sigma = 4$.

- (a) What is the probability that a fisherman catches a spotted sea trout within the legal limits? [2 pts]
- (b) Find x such that the probability that the length of a randomly caught sea trout is less than or equal to x , is 0.5.
Hint: Use the fact that if Z is a standard normal random variable, then $\Phi(0) = P(Z \leq 0) = 0.5$. Now express $X =$ Length of a randomly caught sea trout in terms of Z . [4 pts]
- (c) What is the probability that the fisherman will catch three trout outside the legal limits before catching his first legal spotted sea trout (between 14 and 30 inches)? Assume that all his catches are independent. [4 pts]