STA 4321/5325

Spring 2010

Sample Exam: Joint Distributions

Full Name: _____

On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: _____

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one letter-sized sheet (the same size as the lecture notes) of your own notes and a pocket calculator. (You are not required to bring a calculator — you may leave your answers in a form from which the numerical answer could be immediately calculated.) You may *not* use any books, other references, or text-capable electronic devices.

1. The proportions X and Y of two chemicals found in samples in an insecticide have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 2 & \text{if } 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le x+y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Provide the marginal probability density function of Y at 0.7, i.e., provide $f_Y(0.7)$. [2 pts]

   (b) Provide the conditional probability density function of X given $Y = 0.7$. [4 pts]

   (c) Provide $P(X > 0.95 \mid Y = 0.7)$. [4 pts]

2. Suppose we have two random variables X and Y with joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} e^{-x} & \text{if } 0 \le y \le x < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Provide $P(X < 2, Y > 1)$. [2 pts]

   (b) Provide $P(X \ge 2Y)$. [4 pts]

   (c) Provide the marginal density fucntion of X. [4 pts]

3. Consider X and Y with joint density function as in Problem 1.

   (a) Provide $E[X]$. [2 pts]

   (b) Provide $E[X+Y]$. [4 pts]

   (c) Proivde $V(X+Y)$. [4 pts]

   Hint: Note that $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)dxdy$.

4. Let X and Y be independent random variables with $E[X] = 56, V(X) = 16$, and $E[Y] = 5, V[Y] = 4$.

(a) Provide $Cov(X, Y)$. [2 pts]

(b) Provide $V(X + Y)$. [4 pts]

(c) Provide $V(X - Y)$. [4 pts]