

SOLUTIONS TO SAMPLE EXAM

Problem 1: (a) Let A = Event that chosen Cypriot speaks English.

B_1 = Event that chosen Cypriot is Greek

B_2 = Event that chosen Cypriot is Turkish

By the law of total probability,

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$$

$$= \frac{20}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100}$$

$$= 0.17$$

$$(b) P(B_2|A) = \frac{P(A \cap B_2)}{P(A)}$$

$$= \frac{P(A|B_2)P(B_2)}{P(A)}$$

$$= \frac{\frac{10}{100} \times \frac{30}{100}}{0.17}$$

$$= \frac{3}{17}$$

Problem 2: (a) A typical outcome of this experiment is a 3-tuple xyz where, xyz is a permutation of 123. There are $3! = 6$ ways of permuting the symbols 123. Hence, the sample space of this experiment is

$$S = \{123, 132, 231, 213, 312, 321\}$$

$$(b) \quad A = \{123, 132\}$$

$$B = \{123, 213\}$$

$$C = \{312, 231\}$$

A and B are not disjoint as $A \cap B = \{123\}$

A and C are disjoint

B and C are disjoint

$$(c) \quad P(A \cap B) = P(\{123\}) = \frac{1}{6}$$

$$P(A) = P(\{123, 132\}) = \frac{2}{6} = \frac{1}{3}$$

$$P(B) = P(\{123, 213\}) = \frac{2}{6} = \frac{1}{3}$$

$$P(A)P(B) = \frac{1}{3} \neq \frac{1}{6} = P(A \cap B)$$

Hence A and B are not independent.

Problem 3: See Homework Solutions (Exercise 2.18)

Problem 4: See Homework Solutions (Exercise 3.49)