FSU-UF JOINT TOPOLOGY AND GEOMETRY MEETING, FEBRUARY 11-12, 2022

Friday, February 11th:

- 4:05 - 4:55: Sam Ballas (FSU), Colloquium, Little Hall 225, Title: Complex projective structures on surfaces
- 6:30 – 9:00: Banquet outside the Little Hall building

Saturday, February 12th: all talks in Little Hall 225

- 8:30 - 9:00: Coffee break.
- 9:00 - 9:50: Tom Needham (FSU), Title: Symplectic Geometry and Signal Processing
- 9:55 - 10:25 Patrick Heslin (FSU), Title: Geometric Hydrodynamics
- 10:25 - 10:45: Coffee break.
- 10:45 - 11:10: Iryna Hartsock (UF), Title: Topological and metric properties of spaces of generalized persistence diagrams
- 11:10 - 11:35: Jared Miller (FSU), Title: Conjugating subgroups of $PGL(2,\mathbb{C})$ into $PGL(2,\mathbb{R})$
- 11:35 - 12:00: Alexander Elchesen (UF), Title: Relative Optimal Transport
- 12:05 - 12:35: Ignat Soroko (FSU), Title: Divergence in Coxter Groups
- 12:35 - 1:50: Lunch Break
- 1:50 - 2:20: Anindya Chanda (FSU), Title: Quasigeodesic Anosov Flows in Dimension 3
- 2:20 - 2:45: Emmanuel Hartman (FSU), Title: A Numerical Framework for Shape Analysis Using Split Second-Order Sobolev Metrics

- 2:45 - 3:10: Jonathan Bush (UF), Title: Vietoris—Rips complexes of spheres and generalizations of the Borsuk—Ulam theorem
- 3:10 - 3:30: Coffee Break.
- 3:30 - 4:00: Jamie Scott (UF), Title: : Rudyak's Conjecture and Surgery Theory
- 4:00 - 4:50: Eric Klassen (FSU): Title: Elastic Shape Analysis, Square Root Normal Fields and Unbalanced Optimal Transport

ABSTRACTS:

- **Sam Ballas**: Complex projective structures on surfaces.

  Abstract: Roughly speaking, a complex projective structure on a surface is way of locally identifying the surface with the complex plane in such a way that the transition maps are Mobius transformations. Despite their geometric description, these types of structures have a long history related to solutions to certain second order differential equations going back to the work of Poincare in the early 1900s. In this setting, the solution to the differential equation allows one to construct a complex projective structure. When the surface is closed the space of all such complex projective structures is well studied and understood, however, the non-compact case is still poorly understood. In this talk I give an overview of complex projective structures on surfaces, survey what is known in the closed setting, and describe some recent work (joint with P. Bowers, A. Casella, and L. Ruffoni) where we provide a complete geometric description of a certain slice of the space of projective structures on the thrice punctured sphere.

- **Tom Needham**: Symplectic Geometry and Signal Processing

  Abstract: In this talk, I will describe novel connections between the fields of symplectic geometry and frame theory. Symplectic geometry was originally introduced as a mathematical formalism for classical mechanics and has since developed into its own rich subfield of differential geometry. Frame theory was developed more recently, and involves the study of spanning sets for vector spaces which have favorable robustness properties for signal processing applications. We will show how the symplectic perspective gives new geometric proofs and generalizations of classical results from frame theory, suggesting possibilities for broader applications to machine

learning theory.

- **Patrick Heslin**: Geometric Hydrodynamics

  Abstract: V. Arnold observed in his seminal paper that solutions of the Euler equations for ideal fluid motion can be viewed as geodesics of a certain right-invariant metric on the group of volume-preserving diffeomorphisms (known as volumorphisms). In essence, this approach showcases the natural framework in which to tackle this infamous Cauchy problem from the so-called Lagrangian viewpoint. In their celebrated paper Ebin and Marsden provided the formulation of the above in the H^s Sobolev setting. Here they proved that the space of H^s volumorphisms can be given the structure of a smooth, infinite dimensional Hilbert manifold. They illustrated that, when equipped with a right-invariant L^2 metric, the geodesic equation on this manifold is a smooth ordinary differential equation. They then applied the classic iteration method of Picard to obtain existence, uniqueness and smooth dependence on initial conditions. In particular, the last property allows one to define a smooth exponential map on $D^s_\mu(M)$ in analogy with the classical construction in finite dimensional Riemannian geometry. Hence, the work of Arnold, Ebin and Marsden allows one to explore the problem of ideal fluid motion armed with tools from Riemannian geometry.

- **Iryna Hartsock**: Topological and metric properties of spaces of generalized persistence diagrams

  Abstract: Persistence homology is a tool used in topological data analysis to study the shape of data. It starts with a family of spaces indexed by the real numbers and applies homology with coefficients in a field. The persistence diagram is a complete invariant of the resulting diagram of vector spaces. Persistence diagrams are formal sums in a pair of metric spaces and have a family of canonical metrics called Wasserstein distances. I will discuss the topological and metric properties of the resulting spaces of persistence diagrams. These properties both facilitate and constrain what types of analysis are possible. An advantage of this general approach is that it also applies to "persistence diagram" obtained from families of spaces indexed by multiple parameters.

- **Jared Miller**: Conjugating subgroups of $PGL(2, \mathbb{C})$ into $PGL(2, \mathbb{R})$

  Abstract: Projective transformations acting on $\mathbb{C}P^1$ may or may not be conjugate into $PGL(2, \mathbb{R})$. Since real transformations will preserve the standard copy of $\mathbb{R}P^1$, or equivalently a circle in $\mathbb{C}P^1$,

and matrix similarity preserves eigenvalues, only matrices whose ratio of eigenvalues is real or of magnitude 1 have any chance to be conjugate into $PGL(2, \mathbb{R})$. In this talk we discuss when a finitely generated subgroup in $PGL(2, \mathbb{C})$ is conjugate into $PGL(2, \mathbb{R})$.

- **Alex Elchesen**: Relative Optimal Transport

  Abstract: The Wasserstein distances between persistence diagrams are widely used for quantifying topological dissimilarity of filtered topological spaces. While these distances were inspired by and named for the classical Wasserstein distances between probability measures arising in optimal transport, only recently has the precise connection to the classical theory been studied. Expanding on recent work by Divol and Lacombe, I will describe a general theory of "relative" optimal transport.

  The relative optimal transport problem asks, given two Radon measures μ and ν on a pointed metric space (X, d, x_0), what is the most efficient way to transport the mass of μ to that of ν, allowing mass to be transported to and from the basepoint x_0? The relative Wasserstein distance is the cost of the most efficient transport plan. This generalizes the classical optimal transport theory by allowing comparison of measures of different total mass. The Wasserstein distances between persistence diagrams are then examples of the relative Wasserstein distances. I will also discuss connections to linear functionals on spaces of Lipschitz functions and state a Riesz representation-type theorem relating such functionals to the relative Wasserstein distance.

- **Ignat Soroko**: Divergence in Coxter Groups

  Abstract: Divergence of a metric space is an interesting quasi-isometry invariant of the space which measures how geodesic rays diverge outside of a ball of radius r, as a function of r. Divergence of a finitely generated group is defined as the divergence of its Cayley graph. For symmetric spaces of non-compact type the divergence is either linear or exponential, and Gromov suggested that the same dichotomy should hold in a much larger class of non-positively curved CAT(0) spaces. However this turned out not to be the case and we now know that the spectrum of possible divergence functions on groups is very rich. In a joint project with Pallavi Dani, Yusra Naqvi, and Anne Thomas, we initiate the study of the divergence in the general Coxeter groups. We introduce a combinatorial invariant called the 'hypergraph index', which is computable from the Coxeter graph of the group, and use it to characterize when a Coxeter group has linear, quadratic or exponential divergence, and also when its

divergence is bounded by a polynomial.

- **Anindya Chanda**: Quasigeodesic Anosov Flows in Dimension 3

  Abstract: A flow is called 'Quasigeodesic' if its flowlines are length minimizing up to a multiplicative and/or an additive constant when lifted to the universal cover. As quasigeodesity is a metric invariant property it turns out to be an extremely powerful tool in the study of large scale geometry. In this talk we will first review some old and important examples and results on quasigeodesic Anosov flow and then we will talk about some new examples in this area and their possible applications.

- **Emmanuel Hartman**: A Numerical Framework for Shape Analysis Using Split Second-Order Sobolev Metrics

  Abstract: We present a novel numerical framework to perform shape analysis on the space of shapes with respect to a family of split second-order Sobolev metrics. Our framework uses a varifold matching approach to solve the registration of surfaces without the need to discretize the space of diffeomorphisms or the action on the space of parameterized surfaces. In addition, the framework works directly with simplicial meshes with or without boundary and any genus surface. Finally, the varifold matching approach can be adapted to incorporate weights on the faces of simplicial meshes for partial matching problems. This framework is complete with a statistical framework for computing Karcher means, principle components, and motion transfer.

- **Jonathan Bush**: Vietoris—Rips complexes of spheres and generalizations of the Borsuk—Ulam theorem

  Abstract: The famous Borsuk—Ulam theorem, in one of its many formulations, states that a continuous map from an n-sphere to euclidean n-space must have a zero if the map respects the antipodal action on both spaces. Generalizations of this theorem for maps from spheres to lower-dimensional euclidean spaces have been established by, for example, Gromov, Dyson, and Yang. We will instead consider generalizations of this theorem for maps from spheres to higher-dimensional euclidean spaces, and we will prove that the image of a small diameter subset of the sphere must contain the origin in its euclidean convex hull. As a special case, we will completely describe the situation for maps from the circle to euclidean spaces, that is, we will give sharp lower bounds on the diameter of the aforementioned subsets in terms of the dimension of the codomain. Finally, we will explain how these generalizations follow from an understanding of the homotopy types of Vietoris—Rips complexes defined on spheres.

- **Jamie Scott**: Rudyak's Conjecture and Surgery Theory

  Abstract: Rudyak's conjecture is about a classical homotopy invariant called Lusternik-Schnirelmann (LS) category. In particular, Rudyak's conjecture states that if $f : M \to N$ is a map of degree one between closed manifolds, then $cat(M)$ is at least as big as $cat(N)$.

  In this talk, we'll first review some of the basics of sectional category and surgery theory. Then we'll discuss a generalized version of Rudyak's conjecture for sectional category, and sketch a proof of the main lemma for how surgery theory can be applied to prove certain cases of this conjecture. Finally, we'll discuss applications of this main lemma to the cases of LS category and topological complexity.

- **Eric Klassen**: Elastic Shape Analysis, Square Root Normal Fields and Unbalanced Optimal Transport.

  Abstract: Square Root Normal Fields (SRNFs) were introduced by Jermyn et al. in 2012, and have proved to be a useful way of performing shape analysis of surfaces in $\mathbb{R}^3$. In this talk, I will demonstrate an equivalence between SRNF distance and a form of unbalanced optimal transport known as the Wasserstein-Fisher-Rao (WFR) metric. I will also demonstrate a new method of computing the WFR metric (and hence SRNF distance) that is efficient and significantly more accurate than the previous methods used to compute SRNF distances. This is joint work with Martin Bauer and Emmanuel Hartman.