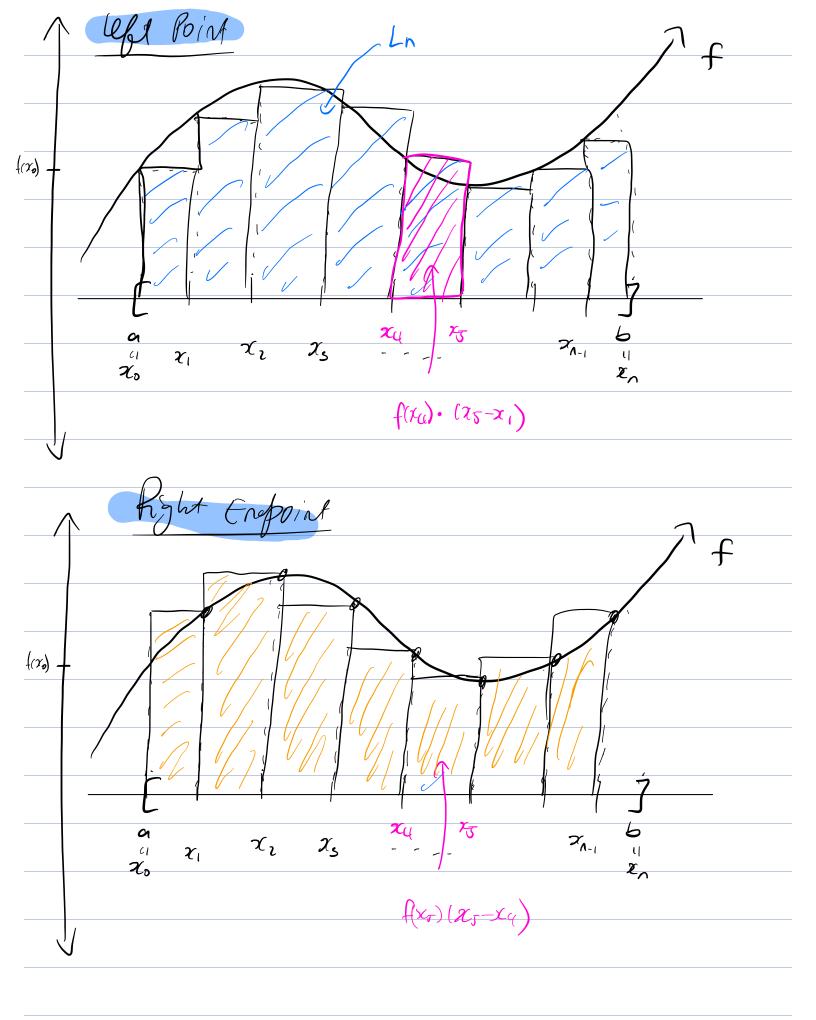
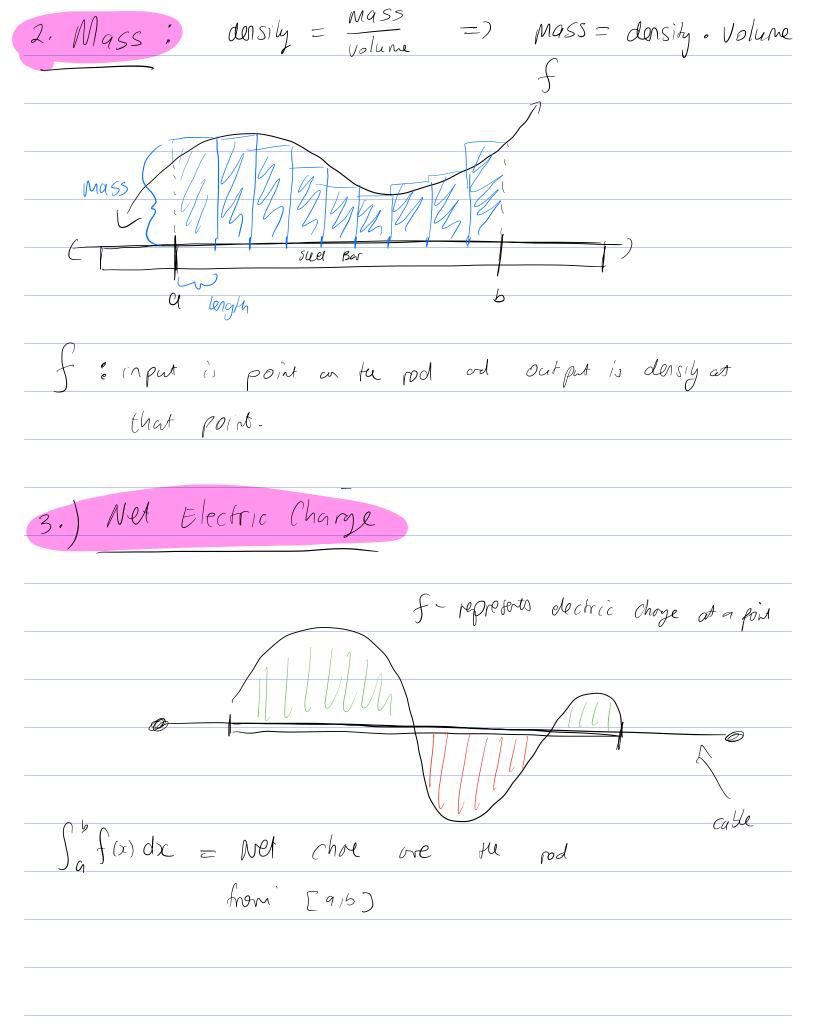
Anti-Derivations and April 9, 2024 Riemann Sums hast time - Applied Ophimization Tod ag - fiemann Sums. - Anti - derivatives. def: An <u>on hi-derivative</u> of a function f(x) is a function F = 5.4 F(x) = f(x). dif: (Riemann Sums) let f: [a15] -> R. left Enpoint i $l_{n} := \sum_{i=0}^{\infty} f(x_i) (x_{i+1} - x_0)$ Right Endpoint $R_n := \sum_{i=1}^{n} f(x_i)(x_i - x_{i-1})$ Summing up are as of $\int M_{\Lambda} f(\mathbf{x}_{i}^{*}) (\mathbf{x}_{i} - \mathbf{x}_{i-1})$ regetangles Midpoin

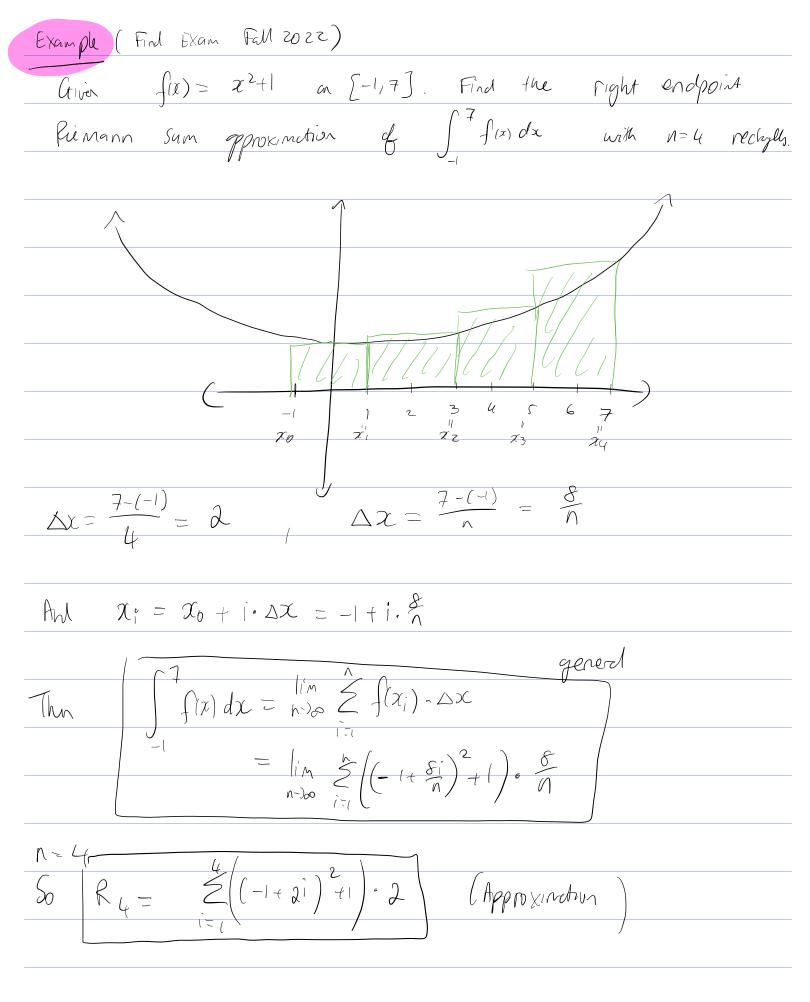


The Are bodynet Gion a cone, how by we find the one inder see Core?		Areas and	Riemann	Sums ?	
Aleman Sun (live files->R, a: x, c = c = c = c = c = c = c = c = c = c	The Area Problem: Given a curve, how	do we find	the area	mder He Cu	rre .
Kiemen Sun blive $f_{1}(z_{1}(z_{2}) \rightarrow R)$, $a = z_{0} < x_{1} < z_{2} = x_{1}$ the Sun $\sum_{i=1}^{n} f_{i}(x_{i}^{-}) \rightarrow x$ wher $z_{i}^{+} \in [2\pi_{i-1}, z_{i}]$ is abled a Remain Sum. $k_{i}gh_{i}^{+}$, $M_{i}dM_{i}$, $k_{i}H^{-}$ end prive approximations: $(z_{i})^{+} e_{i}(z_{i})^{-} x_{i}$, z_{i} , $z_$				F	
the dum $\sum_{i=1}^{n} f(x_i^*) \Delta x$ where $x_i^* \in [X_{i-1}, X_i]$ is culled a Remain Sum. Right, Middle, <u>left-endpoint approximations</u> : (Examples of the mann Sums) $f_n = \sum_{i=1}^{n} f(x_i^*) \Delta x$, $x_i^* = x_i$, $x_{i-1} = x_i^*$ (right out point approximation) $-n = \sum_{i=1}^{n} f(x_i^*) \Delta x$, $x_i^* = x_{i-1}$, $x_{i-1} = x_i^*$ (left out point approximation) $n = \sum_{i=1}^{n} f(x_i^*) \Delta x$, $x_i^* = x_{i-1} = \frac{\Delta x_i}{\alpha}$, $x_i^* = \frac{\Delta x_i}{\alpha}$,	L $ \begin{array}{c} $	A a a Jaz	x_{n-1} $x_n = b$	`	
$\frac{1}{2} \int_{1}^{\infty} \int_{1}^$	iemann Shm, Given f: Ed,63 -> R, a=	ς χ _ο ζ κ ₁ ζ <i>Χ</i> 2 ζ ζ λη	- b		
$L_{n} = \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad , x_{i}^{-1} = x_{i} \qquad x_{i-1} \qquad z_{i} \qquad (fight out four opportuntion)$ $= \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad , x_{i}^{-1} = x_{i-1} \qquad x_{i-1} \qquad z_{i} \qquad (left out four opportuntion)$ $= \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad , x_{i}^{-1} = x_{i-1} + \frac{\Delta x}{x_{i-1}} \qquad x_{i} \qquad (widdle four opportuntion)$ $= \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad , x_{i}^{-1} = x_{i-1} + \frac{\Delta x}{x_{i-1}} \qquad x_{i} \qquad (widdle four opportuntion)$ $= \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad , x_{i}^{-1} = x_{i-1} + \frac{\Delta x}{x_{i-1}} \qquad x_{i} \qquad (widdle four opportuntion)$ $= \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad , x_{i}^{-1} = x_{i-1} + \frac{\Delta x}{x_{i-1}} \qquad x_{i} \qquad (widdle four opportuntion)$ $= \int_{i=1}^{n} f(x_{i} + x_{i}) = \sum_{i=1}^{n} f(x_{i}^{-1}) \Delta x \qquad wher a = x_{0} < x_{1} < x_{2} < \dots < x_{n} = 0$ $= \int_{a}^{b} f(x_{i}) dx \qquad := \int_{a}^{1} f(x_{i}^{-1}) \Delta x \qquad wher \qquad a = x_{0} < x_{1} < x_{2} < \dots < x_{n} = 0$	the sum $\sum_{i=1}^{N} f(x_i^*) \Delta x$ where $x_i^* e$	[x _{i-1} , x _i] is colled	a Riema,	in Sum.	
$2n = \sum_{i=1}^{n} f(x_{i}^{*}) \Delta x \qquad , x_{i}^{*} = x_{i} \qquad x_{i-1} \qquad z_{i} \qquad (right end point oppowerhow)$ $n = \sum_{i=1}^{n} f(x_{i}^{*}) \Delta x \qquad , x_{i}^{*} = x_{i-1} \qquad z_{i} \qquad (logt end point oppowerhow)$ $n = \sum_{i=1}^{n} f(x_{i}^{*}) \Delta x \qquad , x_{i}^{*} = x_{i-1} + \frac{\Delta x}{2} \qquad z_{i-1} \qquad z_{i} \qquad (middle point oppowerhow)$ $n = \sum_{i=1}^{n} f(x_{i}^{*}) \Delta x \qquad , x_{i}^{*} = x_{i-1} + \frac{\Delta x}{2} \qquad z_{i-1} \qquad z_{i} \qquad (middle point oppowerhow)$ $etinite Trilegal$ $f(i)en a finction f: [a,b] \rightarrow f R ke define$ $\int_{a}^{b} f(x_{i}) dx \qquad := \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}^{*}) \Delta x \qquad wher a = x_{0} < x_{1} < x_{2} < \dots < x_{n} = a$ $a partition f Ea/b add$	ight, Middle, left - endpoint approximations: (E)	Xamples of his ma	on Sume)	
$n = \sum_{i=1}^{n} f(x; *) \Delta x$ $i = x_{i-1}$ $x_{i-1} = x_{i-1}$ $x_{i-1} = x_{i-1} + \frac{\Delta x}{2}$ $x_{i-1} = x_{i-1} + \frac{\Delta x}{2$, <i>[x_{i-1}, x_i]</i>	
$n = \sum_{i=1}^{n} f(x_i^*) \Delta x \qquad , x_i^* = x_{i-1} + \frac{\Delta x}{2} \qquad \qquad x_i^* = x_i^* + \frac{\Delta x}{2} \qquad (\text{ widdle point approximation})$ refinite Inlegan: Given a finction $f: [a_ib_j - x_i] R$ we define $\int_{a}^{b} f(x_i) dx \qquad := \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x \qquad where a = x_0 < x_1 < x_2 < \dots < x_n = 0$ $a \text{ partition of } [a_ib_j \text{ ord}]$	$k_n = \sum_{j=1}^{\infty} \int (x_j \cdot) \Delta x \qquad \qquad$	π_{i-i}		(right end poin	t oppoximation)
efinite Integral: Given a function $f: [\alpha_1 \beta_3 \rightarrow \beta R$ we define $\int_{a}^{b} f(x) dx := \lim_{n \to \infty} \sum_{i=t}^{n} f(x_i^*) \Delta x \qquad \text{where} \qquad a = x_0 < \pi_1 \angle x_2 \angle \ldots < x_n = 6$ $a \text{ partision } f = [\alpha_1 \beta_3] \text{ ord}$	$-n = \sum_{i=1}^{n} f(x_i^*) \Delta x \qquad \qquad$	\$ \$\frac{1}{2} = 1	-(2;	(left end point	Gpproxi,mation)
We finite $T_n legal$: Given a function $f: [a,b] \to R$ we define $\int_a^b f(x) dx := \lim_{n \to \infty} \sum_{i=1}^n f(x_i^*) \Delta x \text{when} a = x_0 < x_1 \perp x_2 \perp \dots < x_n = 0$ $a \text{ partision} f Ea,b] ad$	$n = \sum_{i=1}^{n} \int (x_i^*) \Delta x \qquad \qquad j = x_{j-1} + \frac{\Delta x}{2}$	x;-1 a	4	(midule point	OPPOKINGA)
Given a finction $\int : [\alpha_1 \beta_3] \to \beta R$ we define $\int_{a}^{b} f(x) dx := \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x \qquad \text{where} a = x_0 < x_1 < x_2 < \dots < x_n = 6$ $a \text{ partision } f [\alpha_1 \beta_3] \text{ od}$					
Given a finction $\int : [\alpha_1 \beta_3 \to N] R$ we define $\int_{a}^{b} f(x) dx := \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x \qquad \text{where} a = x_0 < x_1 \angle x_2 < \dots < x_n = 6$ $a \text{ partision } f [\alpha_1 \beta_3] \text{ od}$					
a partision of Ea, 63 ord	lefinite Inlegal:				
a partision of Ea, 63 and	Given a function f: [a, b] -> R				
If the limit exists, then we rice Exist, zi]	Given a function f: [a, b] -> R		$Q = x_o < x$	1 ζ χ ₂ ζ < χ _n = ⁶	
	Given a function $\int : [\alpha_{i}, b_{j}] \rightarrow i\mathbb{R}$ $\int_{a}^{b} f(x) dx := \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}, b_{i})$	DX While a part	histon of		

o There are functions that are not Rumann inlegrable. But this need not be a problem since all the functions in this course are continuous, or continuus except at finitely many points: (Special Cute : Lebesgone's Criterion for Riomann Integrability) Thm: If f: Ea, b] -> R continuous, on if f has at most finitely many points of discontinuity, then $\lim_{n\to\infty} \sum_{i=1}^{n} f(\alpha_i) \cdot \Delta \chi$ exists. 3 ways to Visualize Inlegrals Area, Mass, Electric charge $I. Areq : \int_{0}^{5} f(x) dx = \frac{\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x}{\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x}, \quad a = x_0 \angle x_1 \angle \dots \angle x_n = 5.$ height a width. 6



Evaluate the opper and low sum for f(x) = 2+sinx Example: n = 4O,T 4 f 3 l ×3 1 πı Xu=7 *π*0 ~ ∂ $R_{i} = \sum_{j=1}^{4} \int (x_{j}) \cdot \Delta x_{j}$ $\Delta \chi = \frac{\pi - \circ}{4} = \frac{\pi}{4}$ i = I SZ Z $\int (\frac{\pi}{4}) \sim 2 +$ $X_1 = O + \overline{\xi}_1 = \overline{\xi}_1$ $\chi_2 = 0 + 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$ - $\mathcal{I}_3 = 0 + 3 \cdot \frac{7}{4} = \frac{37}{4}$ 2 + Jz Z $\left\{ \left(\begin{array}{c} 37\\ 4 \end{array} \right) \right\} =$ 1 = 0 +4·7 - 1 \mathcal{X}_{ij} $f(\tau) = 2 +$ 0 $R_{4} = \sum_{i=1}^{4} f(i \cdot \frac{\pi}{4}) \cdot \frac{\pi}{4} = (9 + \sqrt{2}) \cdot \frac{\pi}{4}$ then $L_4 = \tilde{\Sigma} f(\chi;) \cdot \Delta \chi = (9+Jz) \cdot \tilde{\Xi}$ 1=0



Example 1: Given $f(x) = x^2 + 1$ on $[o_1 3]$. Use $\sum_{i=1}^{n} \frac{2}{6} = \frac{n(n+i)(n+2)}{6}$ to find the crea order the corve of four [0,3] excot _ · · · xo T_ \mathcal{L}_{i} χ_{1} X, 3 0 $\Delta \chi = \frac{b-a}{n} = \frac{3}{3}$ $x_0 = 0$, $x_1 = \frac{3}{n}$, $x_2 = \frac{6}{n}$, \dots , $x_n = 3$ $\chi_i = \chi_6 + i \cdot \Delta \chi = i \cdot \frac{3}{5}$ $R_n = \sum_{i=1}^{n} f(x_i) \Delta x = \sum_{i=1}^{n} \left(\left(\frac{3i}{n} \right)^2 + 1 \right) \cdot \frac{3}{n}$ $= \left(\left(\frac{q}{a^2} \hat{z}^{(2)} + n \right) \right)^{\frac{3}{n}}$ = 27 x12 + 3 $=\frac{27}{3}\cdot\frac{n\cdot(n+1)(2n+1)}{6}+3$ Then $\int_{-\infty}^{3} f dx = \lim_{n \to \infty} Rn = \lim_{n \to \infty} \frac{27}{n^{3}} \cdot \frac{n \cdot (n+1)(2n+1)}{43}$ $=\frac{27\cdot 2}{6}+3=|12|$

Example 2: Given
$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^2}{n^3} = E \circ H = 1$$

Find a function f for which the limit represents
the area of under the graph. (Eight arguet.)
 $\int_{0}^{1} f dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}) \cdot \Delta x$, $\Delta x = \frac{1}{n}$
 $= \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}) \cdot \frac{1}{n}$, $f(x) = x^{2}$
 $= \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}) \cdot \frac{1}{n}$, $f(x) = x^{2}$
 $= \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i}) \cdot \frac{1}{n}$, $f(x) = x^{2}$
 $= \lim_{n \to \infty} \sum_{i=1}^{n} f(x) dx = \lim_{n \to \infty} \sum_{i=1}^{n} (2 + \frac{1}{n^{2}}) \frac{6}{n}$
 $= \lim_{n \to \infty} \sum_{i=1}^{n} (4 + 2(i \cdot \frac{3}{n})) \frac{3}{n} \cdot 2$
 $= \lim_{n \to \infty} \sum_{i=1}^{n} f(x) dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x) \Delta x$
 $\lim_{n \to \infty} \sum_{i=1}^{n} f(x) = 2x^{2}H$.

Example: Find f(x) given $f'(x) = 1 + 3 J_x$ and the initial condition flue) = 25. Solution : The arti-derivative of $f'(x) = 1 + 3x^{\frac{1}{2}}$ $f(x) = x + 3x^{\frac{3}{2}} + C$ ίj $= \chi + 2\chi^{3/2} + C$ Then $25 = f(4) = 4 + 2(4)^{2} + C$, solve C $= 2 (\sqrt{2})^{3} + C$ =7 21 $= 2 \cdot 2^{3} + C$ -? $C = \frac{2!}{1L}$ This the solution to the above equation is $f(\chi) = \chi + 2\chi^{3/2} + \frac{21}{16}$



Find the anti-derivatives of the following function. Use C to denote the constant.

1.
$$(x^{2}-1)^{2} = x^{4} - 2x^{2} + 1 \quad \longrightarrow \quad x^{5} - \frac{1}{2}x^{5} + x + C$$
2.
$$\sqrt{\frac{3}{z}} = \sqrt{3} z^{\frac{1}{2}} \quad \longrightarrow \quad \sqrt{3} \frac{2^{\frac{1}{2}}}{z} = 2\sqrt{3z} + C$$
3.
$$\frac{4+u^{2}}{u} = \frac{4}{u} + u \quad \longrightarrow \sqrt{4} \ln |u| + u^{2}_{x} + C$$
4.
$$\cos(\theta) \quad \therefore \quad \sin(\theta)$$
5.
$$-2\cos(\theta)\sin(\theta) = -\sin(2\theta) \quad \longrightarrow \quad \frac{\cos(2\theta)}{2} + C$$
6.
$$\cos(\theta)\sin(\theta) = \frac{1}{2}(z\sin\theta(\omega\theta)) = \frac{1}{2}(-\frac{\cos(2\theta)}{2})$$
7.
$$\frac{1}{4}x qC^{\sin(2)}(x)$$
6.
$$e^{2u+1} \quad \longrightarrow \quad \frac{\theta^{2u+1}}{2} + C$$
8.
$$e^{2u+1} \quad \longrightarrow \quad \frac{\theta^{2u+1}}{2} + C$$
9.
$$|x| = \sum_{i=1}^{n} \frac{x_{i}}{x_{i}} x_{i} = \sum_{i=1}^{n} \frac{x_{i}}{x_{i}} = \sum_{i=1}^{n}$$