Discussion Notes 6

Oct 1, 2024

Last Time

- Improper Integrals
- Areas, Volumes.
- Disk, washer method

Today

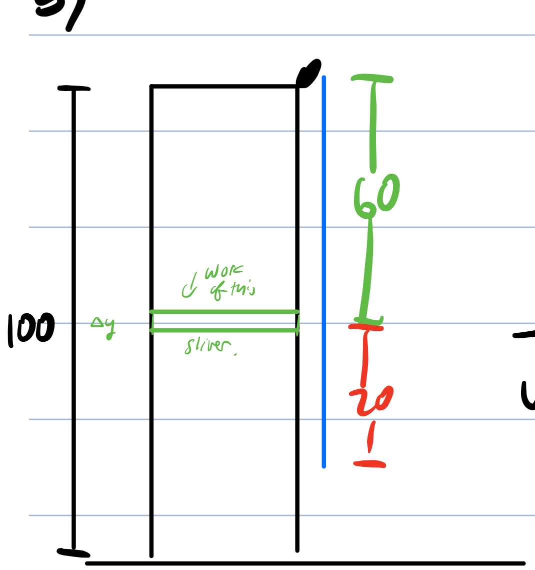
- Probability.
- Work.

Question 1: (Example working with lifting ropes/chains)

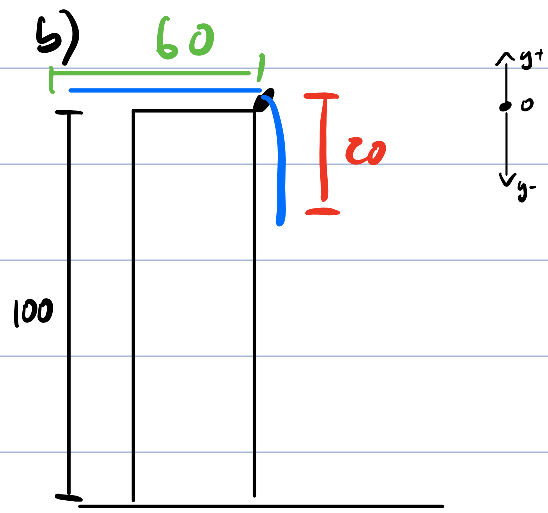
2. An 80 m long rope with a mass density of 3 kg/m hangs over the edge of a tall building that is 100 m high.
(a) How much work is required to pull the entire rope to the top of the building? Express your answer in terms of g.

(b) How much work is required to pull three quarters of the rope (ie. 60 m of the rope) to the top of the building? Express your answer in terms of g.

b)



work need?



(A) Calculate work need for green 60m part.

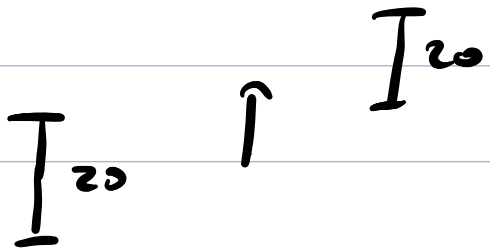
- calculate each sliver, and add up all slivers, i.e integration

- $W_i = F_i d = (Mg)\overset{\text{displacement}}{(-y)} = (\underset{\text{mass}}{\text{length} \cdot \text{density}}) \cdot g \cdot y = (\Delta y \cdot 3) g (-y)$

$$\int_{-60}^{0} -3gy\,dy = \int_{0}^{60} 3gy\,dy = \frac{3g \cdot 60^2}{2}$$

B Calculate work need for green 20 m part.

- This is just the work $W = Fd$, since the mass of the rope stays constant
- Think just moving



• So $W = F \cdot d$

$= m \cdot g \cdot d$ displacement.

$= (20 \cdot 3)\, g \cdot (60)$

length density

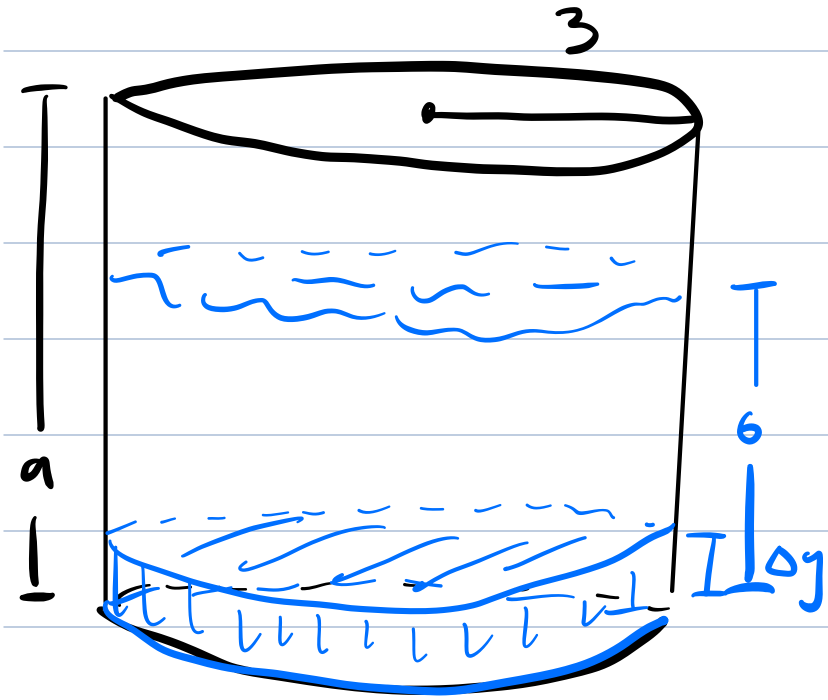
Total work: A + B.

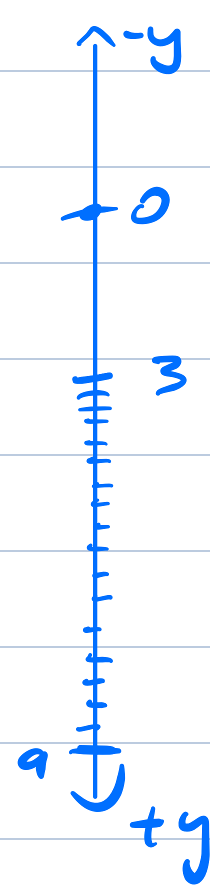
$$\boxed{\frac{3 \cdot g\, 60^2}{2} - 60^2 \cdot g}$$

Example: (Example work with pumping water)

4. A cylindrical tank of radius 3 and height 9 is two-thirds filled with water. How much work is required to pump all the water over the upper rim? Assume that $\rho = g = 1$.

Just to make calculation easier.





$W = F \cdot d$

$= m \cdot g\, y$

$= (\text{density} \cdot \text{volume}) \cdot g \cdot y$

$= (\rho V)\, g \cdot y$

$= 9\pi \cdot \Delta y \cdot \underbrace{\rho g}_{=1} \cdot y$

by assumption

mass = density · volume

Volume

$= \underbrace{\pi \cdot 3^2}_{\text{area}} \cdot \Delta y$ height

324π

$$W = \int_3^9 9\pi \cdot y \, dy$$

$$= \frac{9\pi y^2}{2}\Big|_3^9 = \frac{9\pi}{2}(9^2 - 3^2) = \boxed{324\pi}$$

Example (Example working with springs)

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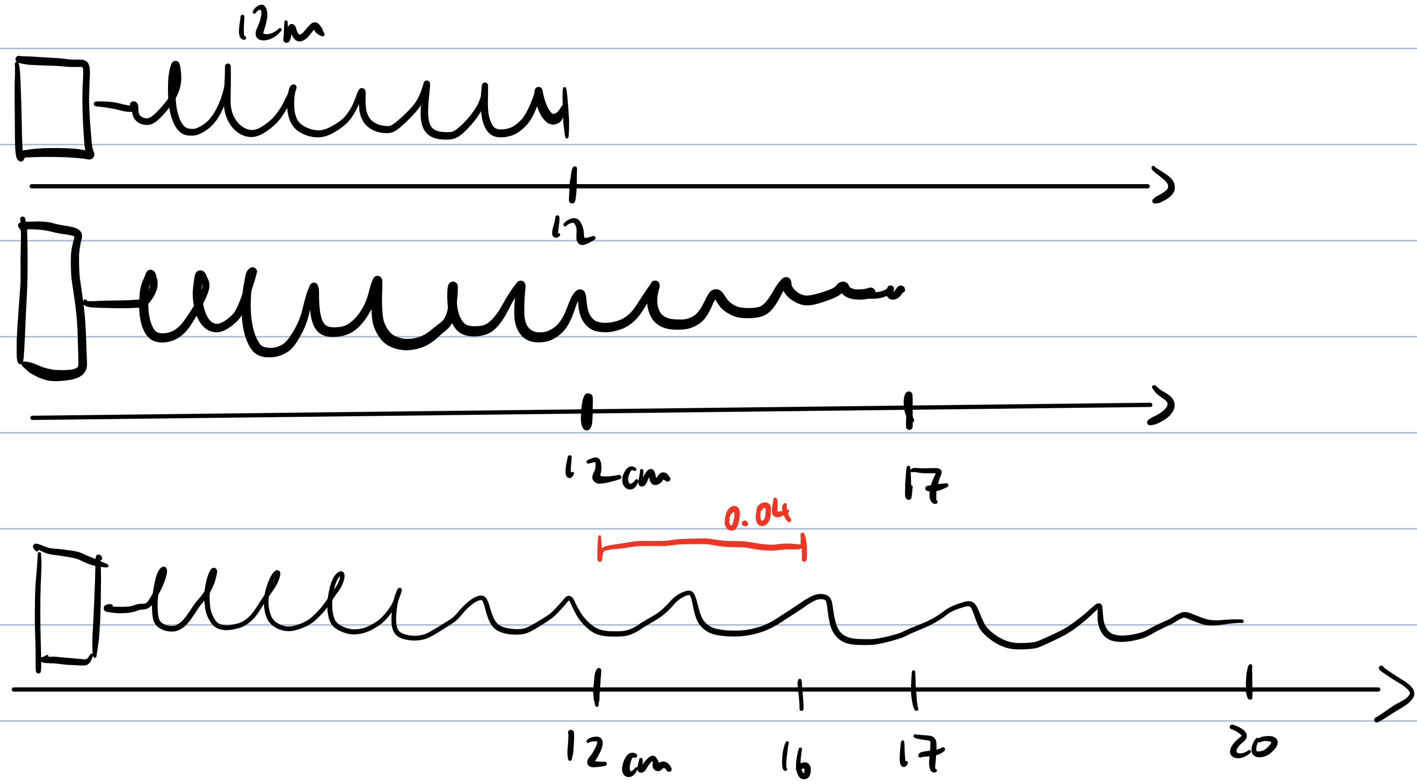
3. A force of 60 N is required to hold a spring that has been stretched from its natural length of 12 cm to 17 cm. How much work is done in stretching the spring from 16 cm to 20 cm?



$F(x) = -kx$, spring constant $k \geq 0$.

Find spring constant. $\swarrow$ $F(x) = -kx$

$\rightarrow$ $60N = k(0.17 - 0.12) \Rightarrow 60 = k\,0.05$

$\Rightarrow k = 6.20$

$\Rightarrow k = 1200$

Work : $\int_{0.04}^{0.08} F(x)\,dx = \int_{0.04}^{0.08} 1200x\,dx$

distance from start

$= \frac{72}{25}.$

ı how much work is requir
blem, $F = -kx$. [Hint: 1
to be converted to meters

Example

2. For what value of c will the function below be a probability density function? (solution: 5/8192)

$$f(x) = \begin{cases} c(8x^3 - x^4) & 0 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad , \quad f(x) \geq 0 \quad \forall x$$

$$\int_0^8 c(8x^3 - x^4)\,dx$$

$$= c\left(\frac{8x^4}{4} - \frac{x^5}{5} \Big|_0^8 \right)$$

$$= c\left(\frac{8 \cdot 8^4}{4} - \frac{8^5}{5} \right)$$

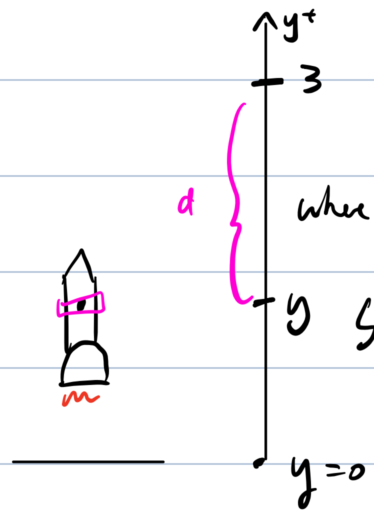
$$= c \cdot \left(\frac{5}{20} \cdot 8^5 - \frac{4}{20} \cdot 8^4 \right)$$

$$= c \cdot \frac{1}{20} \cdot 8^5 = 1$$

$$\Rightarrow \quad c = \frac{20}{8^5}$$

Example :

A rocket weighing 3000 kg is filled with 40000 kg of liquid fuel. In the initial part of the flight, fuel is burned off at a constant rate of 2000 kg per 1000 meters of vertical flight. How much work is required to lift the rocket to 3000 meters? (solution: 120,000,000g J)



mass at y

$M(y) = 3000\,kg + f(y)$

where $f(y)$ = kg fuel at y.

So $f'(y) = -2000 \frac{kg}{km}$

$\Rightarrow f(y) = -2000y + c.$

well $m(y) = -2000y + c + 3000.$

$m(0) = 43\,000\,kg.$

So $m(0) = c + 3000 \Rightarrow c = 40\,000$

$m(y) = -2000y + 43\,000$

$$\int_0^3 m(y)\,g\,dy = \int_0^{3000} (-2000y + 43\,000)\,g\,dy$$

$$= \left(-\frac{2000y^2}{2} + 43000y \Big|_0^{3000} \right) g$$

$$= (-1000 \cdot 3^2 + 43\,000y)\,g$$

$$= (-9\,000\,000 + 129\,000\,000)\,g$$

$$= 120\,000\,000\,g \text{ Jouls.}$$