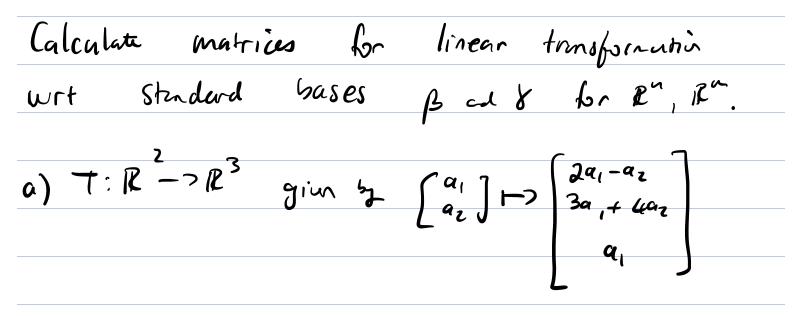
Vomework 4	Our Feb 12, 2025
Solutions and	
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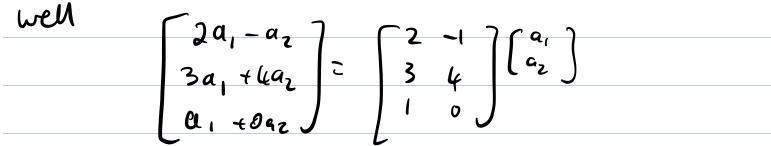
2.2: 1,2(a), 5(c), 10, 17

Grading Scheme . Hw counts 8 morts on Canvas · Spoints per problem · total 25 · So find score is (score). 8 Q1.) 1 Graded Grades QL.) 2(a) Q3.) 5(1) Graded Q4.) 10 ander Q5.) 17 Graded

Question I (FIS, §2.2, nr 1) Griven a vector space V, W, both finite dimensional with ordered bases B at J. And T, U:V-JW linear transformations. a) Tru, a T+u: v->w form a linear transformation b) True, sine $[T]_{\beta}^{\delta} = [U]_{\beta}^{\delta}$ impliès T_{cd} U agree on a basis for V. c) False, $T(v_1) \cdots T(v_m)$ 1NXM d) true, $[T+u]^{\delta} = [T]^{\delta} + [u]^{\delta}_{\beta}$ e) Tru, 2 (V, W) is a vector space f) False $\mathcal{I}(\mathbb{R}^2,\mathbb{R}^3) \neq \mathcal{I}(\mathbb{R}^3,\mathbb{R}^2)$

Queshin 2 (Sz.z, nr Z)





Questin 3 (FIS, SZZ, 5)

c)Griven tr: MZXZ (F) -> F. Comput [tr].

Recall	that	٤	[:], [ه ۱ ر د ن	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	
		-	$M_{i\kappa i}(F)$	•		
basis	for	F.				

The $= \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}$

d) Define $T: P_2(\mathbb{R}) \to \mathbb{R}$ by T(f(x)) = f(z). (orput [7] with respect to EI, x, z ? 3 and E(3. Not [7] must be 1×3.

Well $[T(i) T(x) T(x^2)] = [I Z 4]$

Question 4 (F23, §22, nr 10)
Criver a vector spece V with basis

$$\beta = \Sigma U_{1,...,1} V_{n,3}$$
, Defn $U_{0} := 0$
Recall that then excists a unique lineer $T : U = U$
st $T(U_{j}) = U_{j} + U_{j-1}$. Find $\Sigma T_{j} g$.
Well $\left[[T(U_{1})] [T(V_{2}) - ... [T(V_{n})] \right]$
Well $T(U_{1}) = V_{1} + V_{0} = V_{1} = U_{1} + 2V_{1} + 0V_{2} + ... + 0V_{n}$
 $T(V_{2}) = V_{2} + U_{1} = U_{1} + 2V_{1} + 0V_{3} + ... + 0V_{n}$
 \vdots
 $T(V_{n}) = V_{n} + U_{n-1} = 0V_{1} + ... + V_{n-1} + U_{n}$
So $[T_{j}] = \left[\begin{array}{c} 1 & 1 & 0 & ... & 0 \\ 0 & 1 & 1 & ... \\ 0 & 0 & ... & 1 \end{array} \right]$

Question 5 (FIS: 82.2, nr 17)

Given finite dimensional vector spaces V, W s.t dim (V) = dim (W), and T: V->W a linear transformation. Find besses B and & for U, W respectfully st [7]8 is chiagonal. Ph: Take B= EV1, ..., Un3 a sasis for V. The ET(U, 1, ..., T(Un) 3 is a generating set for R(T). By the replacement applied to G = E T(V, 7, ..., T(Vn) 3, and L=\$ (to equy set is b.2.) we get a subject of G that forms a basis for R(T). By relabelling, let $\xi T(u_1), ..., T(u_k) 3$, k = nse tu basi, obtaind hun above. Now extend &= 27(u1), ..., 7(uk), wkt1, ..., wn 3 to a basi, for w.

reed to change the basis Now be for V. Indeed T(V;) might not be zero for have the following ! KLJEN But we Fix KLjEn, and consider T(V;). -T(Uj) ESpen ZT(U,), ..., T(U) 3 we Sine thus 3 scales of (i), or (i) 5.+ $\tau(v_j) = \tilde{\boldsymbol{Z}}_{ij}^{(j)} \tau(v_i)$ =) $T(v_j - \tilde{\leq} a_i^{(j)} T(v_i)) = 0$. Call $\widetilde{v}_{j} := v_{j} - \widetilde{\Sigma} \alpha_{i}^{(j)} v_{i}$ Now consider a new basis for V namely $\tilde{\beta} := \Sigma V_1, V_2, \dots, V_k, \tilde{V_{k+1}}, \dots, \tilde{V_n} S$

B is linearly To show our new set independent, obsern $a_1 V_1 + \dots + a_k V_k + a_{k+1} \widetilde{V}_{k+1} + \dots + a_n \widetilde{V}_n = 0$ we car rewrite this as a linear consinction of the original EV1,..., Un3 and oppy linear independence to get a1=0 Vi. And hence we have a set of n linearly independent vectors in an M-dim space, thus its spons the space. So p is a basis for V ahr $T(V_1) = I \cdot T(V_1) + O T(V_2) + \dots + O W n$ $T(V_{\rm K}) = 0.T(v_1) + ... + 1.T(v_{\rm K}) + ... + 0w_{\rm N}$ $T(v_{k}) = 0 = 0 T(v_{1}) + \dots + 0 \cdot w_{n}$

$\left[- \right]^{\beta}$	1	\bigcap	0	0	0		Ð	7	
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2 point	ts for busis on V
2 point	for busis an W
l point	writig matrix of the linear transf.
	Ŭ Ŭ

Method 2 : Given a lineor transformation T:V->W between finite dimension vector spaces dn(w) = din(w).let (k=din(ker(7))) and (ℓ_1, \ldots, ℓ_k) basis for kr(7). By the replacement theorem we can extend to a basis p:= EUI, ..., Un 3 for V. Claim: ET(VKH),..., T(VK) 3 is a basis

br P(T),

First abserve that sine EVI,..., Vn3 spen v, we know $(f_{0}, \xi_{1}, \xi_{2}, \xi_{3}, \xi_{3}$ Sne T(U,) = ... = T(Uk) = 0 we get Spn & T(VK+1), ..., T(Un) 3 = R(7).

linear independence suppose For $\hat{z} \propto T(v_i) = 0.$ 12641

=> T(Ed; T(vi)) = 0 i= K+1 =) Édivi EKer(7). Sine EU, ..., VK3 1=16+1 basis for ker(7) we get 1=10+1 =7 - ZB:V: + ZdiV: =0. Appy linear independen of EV.,..., Un) to conclude di=0 Vi=K+1,...,n, Or reeded to conclud ET(Uk+1), ..., T(V_n)) basis for the R(T). Extend 8:= 2 T(UK+1), ..., T(Un), w, ..., WK 3 to basis for W The $T(v_1) = 0$ = 07(V1) + ... + 0W1 + ... + 0WA $T(V_k) = 3$ $T(U_{k+1}) = T(U_{k+1})$



