Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 6, October 12, 2023

(5 points)

Problem 1 Find the derivative of the function y defined implicitly in terms of x as

$$x^y = y^x.$$

$\ln(x^y) = \ln(y^x)$ (1 point)

$y\ln(x) = x\ln(y)$ (1 point)

$\frac{d}{dx}\left(y\ln(x)\right) = \frac{d}{dx}\left(x\ln(y)\right)$

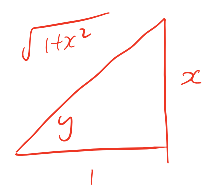
$y'\ln(x) + y\cdot\frac{1}{x} = \ln(y) + \frac{x}{y}\cdot y'$ (1 point - product rule) (1 point - implicit diff)

$y'\ln(x) - y'\cdot\frac{x}{y} = \ln(y) - \frac{y}{x}$

$y'\left(\ln(x) - \frac{x}{y}\right) = \ln(y) - \frac{y}{x}$

$y' = \dfrac{\ln(y) - \frac{y}{x}}{\ln(x) - \frac{x}{y}}$ (1 point)

$\tan^{-1}(x) = y \iff \frac{x}{1} = \tan(y)$



$\frac{d}{dx}\tan(x) = \sec^2 x$

$\cos(y)^2 = \frac{1}{1+x^2}$

Problem 2 .

(1 point)

a.) Given the function

$$f(x) = \tan^{-1}(x). \quad y = \tan^{-1}(x)$$

Find the derivative of f with respect to x.

$\tan(\tan^{-1}(x)) = x$

$\tan(y) = x$

$\frac{d}{dx}(\tan(y)) = \frac{d}{dx}x$

$\sec^2(y)\cdot y' = x$

$y' = \frac{1}{\sec^2(y)}$

$= \cos^2(y)$

$= \frac{1}{1+x^2}$

A.) $f'(x) = \frac{-1}{1+x^2}$

B.) $f'(x) = \frac{1}{\sqrt{1-x^2}}$

C.) $f'(x) = \frac{-1}{\sqrt{1-x^2}}$

D.) $f'(x) = \frac{1}{1+x^2}$

E.) $f'(x) = \frac{-1}{x\sqrt{1-x^2}}$

(3 points)

b.) Find the derivative of the function y defined implicitly in terms of x as

$$y^2 - \tan^{-1}(x) = xy.$$

That is find y':

$$\frac{d}{dx}\left(y^2 - \tan^{-1}(x)\right) = \frac{d}{dx}(xy)$$

$$2y\cdot y' - \frac{1}{1+x^2} = y + xy'$$

$$2y\cdot y' - xy' = y + \frac{1}{1+x^2}$$

$$y'(2y - x) = y + \frac{1}{1+x^2}$$

$$y' = \frac{y + \frac{1}{1+x^2}}{2y - x}$$

(1 point)

c.) Given that $y(0) = 1$ find $y'(0)$.

$$y'(0) = \frac{y(0) + \frac{1}{1+0}}{2y(0) - 0} = \frac{1+1}{2(1)} = 1.$$

(y is a function of x.)