Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 4, September 28, 2023

Problem 1 Given the function

$$f(x) = \frac{e^x}{x^2},\ x \neq 0$$

use the quotient rule and product rule to find:

(3 points)
a.) The first derivative of f with respect to x evaluated at 1. That is find $f'(1)$:

$$f'(x) = \frac{(e^x)(x^2) - (e^x)(2x)}{(x^2)^2} \quad \text{(1 point)}$$

$$= \frac{(e^x)(x-2)(x)}{x^4}$$

$$= \frac{e^x(x-2)}{x^3} \quad \text{(1 point)}$$

$$f'(1) = \frac{e(1-2)}{1^3} = -e \quad \text{(1 point)}$$

(3 points)
b.) The second derivative of f with respect to x evaluated at 1. That is find $f''(1)$:

- If f' incorrect, but $f''(1)$ correctly found from the incorrect f', full credit for part b.

- $$f''(x) = \frac{d}{dx}(f'(x))$$

$$= \frac{d}{dx}\left(\frac{e^x(x-2)}{x^3}\right)$$

$$= \frac{\frac{d}{dx}(e^x \cdot (x-2)) \cdot x^3 - e^x(x-2)(3x^2)}{x^6} \quad \text{(1 point, applying quotient rule again)}$$

$$= \frac{(e^x(x-2) + e^x \cdot (1))x^3 - 3e^x(x-2)(x^2)}{x^6} \quad \text{(1 point, correctly applying product rule)}$$

$$= \frac{(xe^x - 2e^x + e^x)(x) - 3xe^x + 6e^x}{x^4}$$

$$= \frac{x^2e^x - xe^x - 3xe^x + 6e^x}{x^4}$$

$$= \frac{(x^2 - 4x + 6)e^x}{x^4}$$

$$f''(1) = \frac{((1)^2 - 4(1) + 6)e^{(1)}}{(1)^4}$$

$$= (1 - 4 + 6)e$$

$$= 3e \quad \text{(1 point)}$$

$$\begin{array}{r} 5\cancel{6}^{1}4 \\ -27 \\ \hline \end{array}$$

Problem 2 The volume of a growing spherical cell

$$V(r) = \frac{4\pi r^3}{3}$$

where r is the radius of the cell in micro-meters ($1\mu m = 10^{-6}$m).

(2 points)

a.) Find the **average** rate of change of the volume of the cell with respect to change in the radius when the radius changes from $3\mu m$ to $4\mu m$:

$$V_{ave} = \frac{V(r_f) - V(r_i)}{r_f - r_i} = \frac{\frac{4\pi(4)^3}{3} - \frac{4\pi(3)^3}{3}}{1} = \frac{4\pi}{3}(4^3 - 3^3) = \frac{4\pi}{3}(64 - 27)$$

(1 point)

$$= \frac{4\pi}{3}(37)$$

$$= \frac{148\pi}{3} \quad \text{(1 point)}$$

$$\begin{array}{r} 5\cancel{6}^{1}4 \\ -27 \\ \hline 37 \end{array} \qquad \begin{array}{r} {}^{2}37 \\ \times 4 \\ \hline 148 \end{array}$$

(2 points)

b.) Find the **instantaneous** rate of change of the volume of the cell with respect to change in the radius when the radius is $4\mu m$:

$$V_{inst} = V'(r) = 3\left(\frac{4\pi}{3}r^2\right) = 4\pi r^2 \qquad \text{(1 point)}$$

And $V'(4) = 4\pi(4)^2 = 64\pi.$ (1 point)