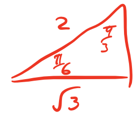
Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I

Quiz # 8, October 26, 2023

$\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$



$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin(2\pi) = 0$

(2 points) **Problem 1** Given $y = t + sin(t) + 2$ as t goes from 2π to $\frac{13\pi}{6}$.

x_1 x_2

a.) Calculate Δy:

$\Delta y = y(x_2) - y(x_1)$

$= \frac{13\pi}{6} + \sin\left(\frac{13\pi}{6}\right) + 2 - \left(2\pi + \sin(2\pi) + 2\right)$ (1 point)

$= \frac{13\pi}{6} + \frac{1}{2} - 2\pi$

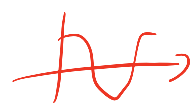
$= \frac{\pi}{6} + \frac{1}{2}$ (1 point)

$= \frac{\pi + 3}{6}$

(2 points) b.) Calculate dy:

$y' = 1 + \cos(t)$ (1 point)

$\cos(2\pi)$



$dy = y' dx \approx y'(x_1) \cdot \Delta x$

$= \left(1 + \cos(2\pi)\right) \cdot \frac{\pi}{6}$

$= \frac{2\pi}{6}$

$= \frac{\pi}{3}$ (1 point)

$\Delta x = x_2 - x_1$

$= \frac{13\pi}{6} - \frac{12\pi}{6} = \frac{\pi}{6}$

(6 points)

Problem 2 .

Given

$$f(x) = e^{-x} - e^{-2x}.$$

Find the absolute minimum, and absolute maximum of f on the interval $[0, \ln(3)]$.

① Local extrema on $(0, \ln(3))$

$f'(x) = -e^{-x} + 2e^{-2x}$

- f' exists for every x in $(0, \ln(3))$

Set $f'(c) = 0$ (1 point)

So $-e^{-c} + 2e^{-2c} = 0$

$\Rightarrow (-e^{-c})(1 - 2e^{-c}) = 0$

never 0 then $1 - 2e^{-c} = 0 \Rightarrow e^{-c} = \frac{1}{2}$

$\Rightarrow \ln(e^{-c}) = \ln(\frac{1}{2})$

$\Rightarrow -c\ln(e) = \ln(\frac{1}{2})$

$\Rightarrow c = -\ln(\frac{1}{2}) > 0$, $-\ln(\frac{1}{2}) = \ln(2)$

$f(-\ln(\frac{1}{2})) = e^{\ln(\frac{1}{2})} - e^{2\ln(\frac{1}{2})}$

$= \frac{1}{2} - \frac{1}{2^2}$

$= \frac{2}{4} - \frac{1}{4}$

$= \boxed{\frac{1}{4}}$

② End points

$f(0) = e^{0} - e^{-2(0)} = 1 - 1 = \boxed{0}$ (1 point)

$f(\ln(3)) = e^{-\ln(3)} - e^{-2\ln(3)}$

$= e^{\ln(3^{-1})} - e^{\ln(3^{-2})}$

$= 3^{-1} - 3^{-2}$

$= \frac{1}{3} - \frac{1}{9}$

$= \frac{3}{9} - \frac{1}{9} = \boxed{\frac{2}{9}}$ (1 point)

x	$f(x)$
0	0
$-\ln(\frac{1}{2})$	$\frac{1}{4}$
$\ln(3)$	$\frac{2}{9}$

- f has a absolute maximum value of $\frac{1}{4}$ at the point $-\ln(\frac{1}{2})$ (1 point)
- f has a absolute minimum value of 0 at the point 0 (1 point)

Optional feedback. Please provide any comments or feedback on how the discussion classes are going. Anything you would like to see done differently? Are there aspects that are working well and that you would like to see more of? Or anything that you think will benefit your success in the course?