Discussion Class October 5, 2023

Last time:

- Product rule, quotient rule, higher order derivatives
- Application

Today:

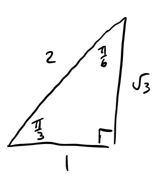
- Derivatives of trigonometric functions
- Chain Rule.

Example 1:

Given a function $f(x) = \dfrac{1+\sec(x)}{1-\sec(x)}$ find $f'(\frac{\pi}{3})$.

$$f'(x) = \frac{\frac{d}{dx}(1+\sec(x)) \cdot (1-\sec(x)) - (1+\sec(x)) \cdot (\frac{d}{dx}(1-\sec(x)))}{(1-\sec(x))^2}$$

$$= \frac{\sec(x)\tan(x)(1-\sec(x)) - (1+\sec(x))(-\sec(x)\tan(x))}{(1-\sec(x))^2}$$



$$= \frac{\{\sec(x)\cdot\tan(x)\}\left[1-\cancel{\sec(x)}+(1+\cancel{\sec(x)})\right]}{(1-\sec(x))^2}$$

$\cos(\frac{\pi}{3}) = \frac{1}{2}$

$\sec(\frac{\pi}{3}) = 2$

$\tan(\frac{\pi}{3}) = \sqrt{3}$

$$= \frac{\sec(x)\cdot\tan(x)\cdot 2}{(1-\sec(x))^2}$$

$$= \frac{2\cdot\sqrt{3}\cdot 2}{(1-2)^2}$$

$$= \boxed{4\sqrt{3}}$$

Example 2: Given $f(x) = x^{\ln(x)}$. $(x>0)$

Find the equation of the line tangent to the graph of f at $x=e$.

Note $f(x) = x^{\ln(x)}$

$= e^{\ln(x^{\ln(x)})}$, e^x and $\ln(x)$ are inverses of each other.

$= e^{\ln(x)\cdot\ln(x)}$, $\ln(a^b) = b\cdot\ln(a)$,

$= e^{(\ln(x))^2}$

So $f'(x) = \frac{d}{dx}\left(e^{\ln(x)^2}\right)$

$= e^{\ln(x)^2}\cdot\frac{d}{dx}\left(\ln(x)^2\right)$, chain rule

$= e^{\ln(x)^2}\cdot\left(2\ln(x)\cdot\frac{1}{x}\right)$, $\frac{d}{dx}\ln(x) = \frac{1}{x}$

$f'(e) = e^{\ln(e)^2}\cdot 2\ln(e)\cdot\frac{1}{e}$

$= e\cdot 2\cdot\frac{1}{e}$, $\ln(e)=1$

$= 2$ (slope.)

$y_1 = f(e) = e^{\ln(e)} = e$. So $(x_1, y_1) = (e, e)$

So $y - y_1 = m(x - x_1)$

$\Rightarrow y - e = 2(x - e)$

$\Rightarrow \boxed{y = 2x - e}$

Example 3: Given $f(x) = \sqrt{\frac{x+2}{x+1}}$. Find $f'(0)$

$$f'(x) = \frac{d}{dx}\left(\frac{x+2}{x+1}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{x+2}{x+1}\right)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left(\frac{x+2}{x+1}\right)$$, chain rule.

$$= \frac{1}{2}\left(\frac{x+2}{x+1}\right)^{-\frac{1}{2}} \cdot \frac{(1)(x+1)-(x+2)(1)}{(x+1)^2}$$, quotient rule.

$$= \frac{1}{2}\left(\frac{x+2}{x+1}\right)^{-\frac{1}{2}} \cdot \frac{-1}{(x+1)^2}$$

$$f'(0) = -\frac{1}{2}\left(\frac{2}{1}\right)^{-\frac{1}{2}}$$

$$= -\frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \boxed{\frac{-1}{2\sqrt{2}}}$$

, $(2)^{-\frac{1}{2}}$

$= (\sqrt{2})^{-1}$

$= \frac{1}{\sqrt{2}}$