Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I

Quiz # 10, March 26, 2024

(4 points)

Problem 1 Evaluate the following limit using L'Hospital rule,

$$\lim_{x\to 1}\frac{1-x+\ln(x)}{1+\cos(\pi x)}. \qquad \text{"}\frac{0}{0}\text{"}$$

$\ln(1) = 0$

$\cos(\pi) = -1$

Apply L'Hospital to get

$$\lim_{x\to 1}\frac{1-x+\ln(x)}{1+\cos(\pi x)}$$

$$\overset{L'H}{=}\lim_{x\to 1}\frac{-1+\frac{1}{x}}{-\sin(\pi x)\pi} \qquad \text{"}\frac{0}{0}\text{"}$$

Apply L'Hospital again

$$\overset{L'H}{=}\lim_{x\to 1}\frac{-\frac{1}{x^2}}{-\cos(\pi x)\cdot\pi^2} \qquad -\cos(\pi)=1$$

$$=\boxed{\frac{-1}{\pi^2}}$$

(6 points)

Problem 2 .

Given a function f on the interval $[0,2\pi]$ defined as

$$f(x) = \cos(x) + \sin(x).$$

Find the intervals where:

1.) f is increasing $(f' > 0)$

2.) f is concave down $(f'' < 0)$

$f'(x) = -\sin(x) + \cos(x)$

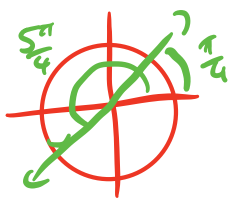
Solve $f'(x) > 0$

Set $-\sin(x) + \cos(x) = 0$

$\Rightarrow \sin(x) = \cos(x)$

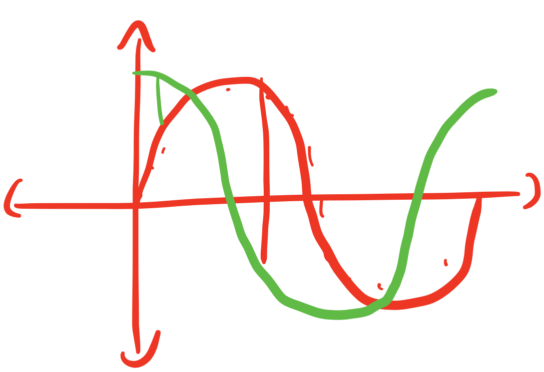
$\Rightarrow \tan(x) = 1$

$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$



	$[0, \frac{\pi}{4})$	$(\frac{\pi}{4}, \frac{5\pi}{4})$	$(\frac{5\pi}{4}, 2\pi]$
f'	$+$	$-$	$+$

Increasing on $[0, \frac{\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi]$



$f''(x) = -\cos(x) - \sin(x)$

Solve $f''(x) < 0$

Set $-\cos(x) - \sin(x) = 0$

$\Rightarrow \cos(x) + \sin(x) = 0$

$\Rightarrow \sin(x) = -\cos(x)$

$\Rightarrow \tan(x) = -1$

$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

	$[0, \frac{3\pi}{4})$	$(\frac{3\pi}{4}, \frac{7\pi}{4})$	$(\frac{7\pi}{4}, 2\pi]$
f''	$-$	$+$	$-$

Concave down on $[0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi]$