Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I

Quiz # 10, November 9, 2023

(3 points)

Problem 1 .

$\frac{d}{dx}(1+x)^{\frac{1}{2}} = \frac{1}{2}(1+x)^{-\frac{1}{2}}$

Evaluate the following limit:

$$\lim_{x\to 0} \frac{\sqrt{1+x} - \frac{x}{2} - 1}{x^2}$$

"$\frac{0}{0}$"

Hint: Use the L'Hospital rule.

$\overset{L'H}{=} \lim_{x\to 0} \dfrac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}}{2x}$ (1 point)

$\overset{L'H}{=} \lim_{x\to 0} \dfrac{\frac{-1}{4}(1+x)^{-\frac{3}{2}}}{2}$ (1 point)

$= \dfrac{\frac{-1}{4}(1+0)^{-\frac{3}{2}}}{2} = \boxed{\frac{-1}{8}}$ (1 point)

(7 points)

Problem 2 .

Given

$$f(x) = \frac{e^x}{x^2},\ f'(x) = \frac{e^x(x-2)}{x^3},\ \text{and}\ f''(x) = \frac{e^x(x^2-4x+6)}{x^4}$$

A.) What is the domain of f? (1 point)

$D(f) = (-\infty, 0) \cup (0, \infty)$

B.) What are the vertical and horizontal asymptotes (if any) of f?

$\lim_{x\to\infty} \frac{e^x}{x^2} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{LH}{=} \lim_{x\to\infty} e^x = \infty$

$\lim_{x\to-\infty} \frac{e^x}{x^2} = 0$ (horizontal asymptote.) (1 point)

$\lim_{x\to 0^+} \frac{e^x}{x^2} = +\infty$

(vertical asymptotes) (1 point)

$\lim_{x\to 0^-} \frac{e^x}{x^2} = +\infty$

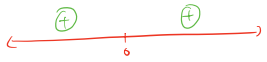
C.) What are the intervals on which f is increasing and decreasing (if any)?

$f'(x) = \frac{e^x(x-2)}{x^3}$

- Increasing on $(-\infty, 0) \cup (2, \infty)$ (1 point)
- Decreasing on $(0, 2)$ (1 point)
- Local min at 2

D.) What are the intervals where f is concave up and concave down (if any)?

$f''(x) = \frac{e^x(x^2-4x+6)}{x^4}$



- Concave up on $(-\infty, 0) \cup (0, \infty)$ and no inflection points

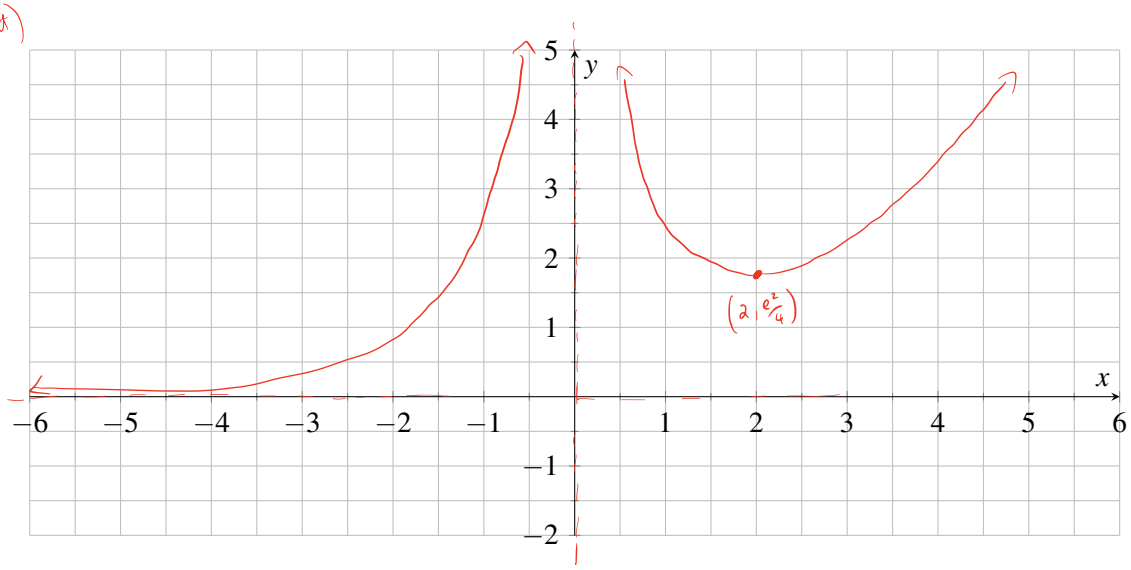
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{(16-4(1)(6)}}{2} = \frac{4 \pm \sqrt{-10}}{2}$

(so no real roots)

(1 point)

E.) Sketch the graph of f using the information from parts A to D, and indicate any local or global extreme values of f if they occur.

(1 point)



$f(2) = \frac{e^2}{4}$