Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 11, April 9, 2024

Problem 1 Find the function g given

$$g'(x) = \frac{(\sqrt{x}-1)^2}{x}$$

with initial condition $g(1) = 2$.

To find the anti derivative $g'(x) = \dfrac{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-1)}{x} = \dfrac{x - 2x^{\frac{1}{2}} + 1}{x}$

$= 1 - 2x^{-\frac{1}{2}} + x^{-1}$

Then we have $g(x) = x - \dfrac{2x^{\frac{1}{2}}}{(\frac{1}{2})} + \ln|x| + C$

$= x - 4\sqrt{x} + \ln|x| + C$

Then to solve C, $2 = g(1) = (1) - 4(1) + \cancel{\ln|1|}^{\,0} + C$

$\Rightarrow 2 = -3 + C$

$\Rightarrow C = 5$

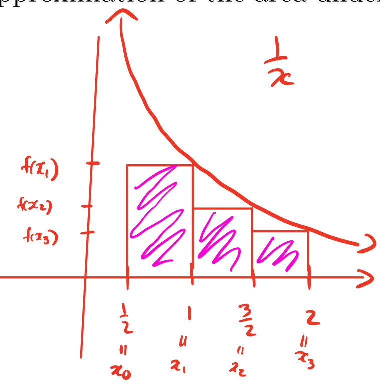
Thus the solution is

$$\boxed{g(x) = x - 4\sqrt{x} + \ln|x| + 5}$$

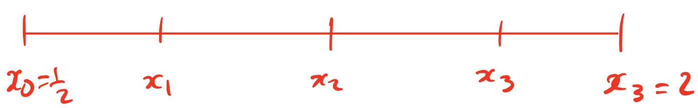
Problem 2 .
Given a function f on the interval $[\frac{1}{2}, 2]$ defined as

$$f(x) = \frac{1}{x}$$

use a Riemann sum with 3 rectangles to find the **right-endpoint** approximation of the area under the graph of f over the interval $[\frac{1}{2}, 2]$.



- $n = 3$
- $\Delta x = \frac{b-a}{n} = \frac{2-\frac{1}{2}}{3} = \frac{\frac{4}{2}-\frac{1}{2}}{3} = \frac{3}{2} \cdot \frac{1}{3} = \frac{1}{2}$
- $x_i = x_0 + i \cdot \Delta x \quad , i = 1, 2, 3$
- $x_0 = \frac{1}{2}$



$$R_3 = \sum_{i=1}^{3} f(x_i) \Delta x$$

$$= \left(f(x_1) + f(x_2) + f(x_3) \right) \frac{1}{2}$$

$$= \left(1 + \frac{2}{3} + \frac{1}{2} \right) \frac{1}{2}$$

$$= \left(\frac{6}{6} + \frac{4}{6} + \frac{3}{6} \right) \frac{1}{2}$$

$$= \boxed{\frac{13}{12}}$$

i	x_i	$f(x_i)$
1	1	1
2	$3/2$	$\frac{2}{3}$
3	2	$\frac{1}{2}$