Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I

Quiz # 12, November 30, 2023

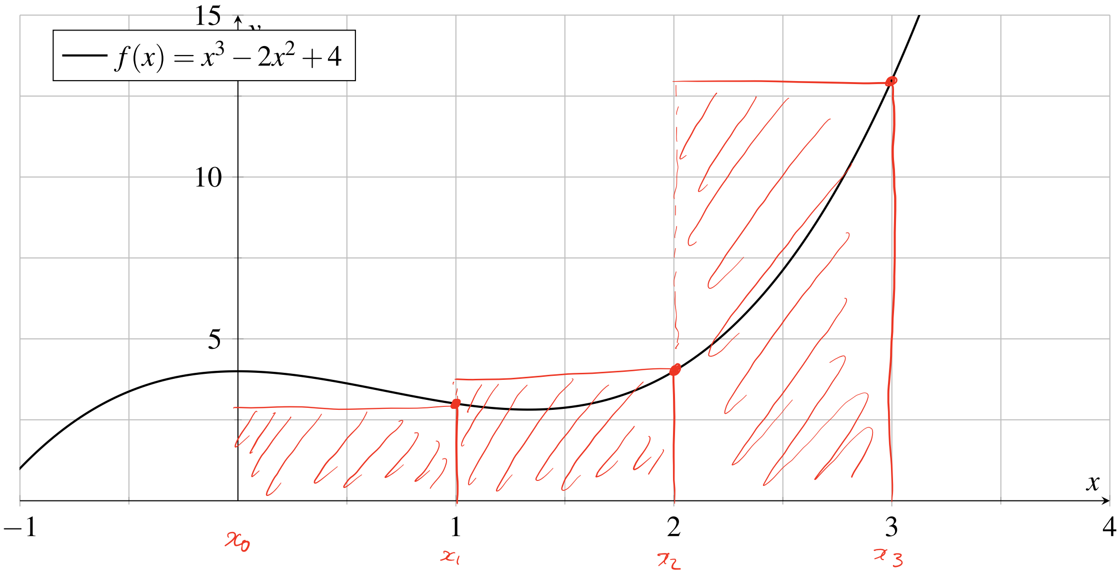
Problem 1 .

Given a function

$$f(x) = x^3 - 2x^2 + 4.$$

We want to approximate the area under the function over the interval [0,3].

(1 point) a.) Sketch the rectangles that we will use to approximate the area using the right-end point rule.



(2 points) b.) Approximate the area under the curve over the interval [0,3] by subdividing the interval into $n = 3$ sub-intervals using the right-end points.

$$\Delta x = \frac{3-0}{3} = 1$$

$x_0 = 0$ $\quad f(x_1) = 1^3 - 2(1)^2 + 4 = 3$

$x_1 = 1$ $\quad f(x_2) = 2^3 - 2(2)^2 + 4 = 4$

$x_2 = 2$ $\quad f(x_3) = 3^3 - 2(3)^2 + 4 = 27 - 18 + 4 = 31 - 18 = 13$

$x_3 = 3$

$$\sum_{i=1}^{3} f(x_i) \cdot \Delta x = f(x_1) + f(x_2) + f(x_3) = 3 + 4 + 13 = \boxed{20}$$

(2 points) c.) Approximate the area under the curve over the interval [0,3] by subdividing the interval into $n = 3$ sub-intervals using the left endpoints.

$\Delta x = \frac{3-0}{1}$

$x_0 = 0$ $\quad f(x_0) = 4$

$x_1 = 1$ $\quad f(x_1) = 3$

$x_2 = 2$ $\quad f(x_2) = 4$

$x_3 = 3$

Then $\displaystyle\sum_{i=1}^{3} f(x_i)\Delta x = 4+3+4 = \boxed{11}$

(1 point) d.) Given the exact area as $\int_0^3 f(x)dx = \frac{57}{4}$ which approximation is closer to the exact value?

$20 = \frac{80}{4}$ $\quad \left|\frac{80}{4} - \frac{57}{4}\right| = \frac{23}{4}$

$11 = \frac{44}{4}$ $\quad \left|\frac{44}{4} - \frac{57}{4}\right| = \frac{13}{4} < \frac{23}{4}$ so L_3 is a better approximation.

Problem 2 .

(4 points)

Evaluate the following integral. Given a Riemann integrable function $f : \mathbb{R} \to \mathbb{R}$ and

$$\int_{12}^{-10} f(x)dx = 6, \quad \int_{100}^{-10} f(x)dx = -2, \text{ and } \int_{100}^{-5} f(x)dx = 4.$$

Use properties of the indefinite integral to calculate

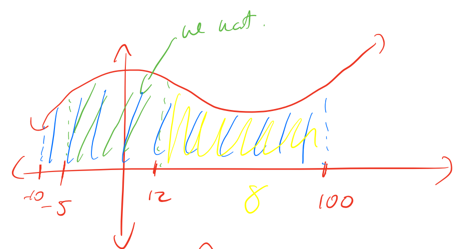
$$\int_{-5}^{12} f(x)dx.$$

① Get $\displaystyle\int_{12}^{100} f\,dx = \int_{-10}^{100} f\,dx - \int_{-10}^{12} f\,dx$

$\displaystyle = -\int_{100}^{-10} f\,dx + \int_{12}^{-10} f\,dx$

$= -(-2) + 6$

$= 8$



Picture represents ⊕ area but only used to find correct endpoints.

② $\displaystyle\int_{-5}^{12} f\,dx = \int_{-5}^{100} f\,dx - \int_{12}^{100} f\,dx$

$\displaystyle = -\int_{100}^{-5} f\,dx - 8 = -(4) - 8 = \boxed{-12}$