Name: Solutions.

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 12, April 16, 2024

Problem 1 .

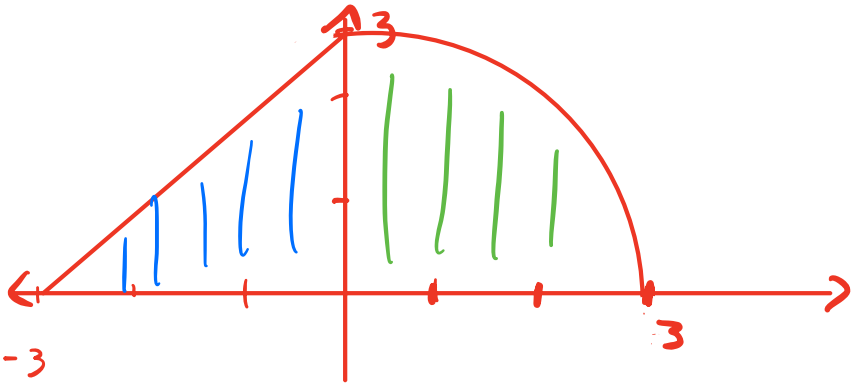
Given

$$f(x) = \left\{ \begin{array}{ll} x+3, & \text{if } x < 0 \\ \sqrt{9-x^2}, & \text{if } x \geq 0 \end{array} \right\} \tag{1}$$

Use geometry to calculate the integral

$$\int_{-3}^{3} f(x)dx$$

Hint: Think triangles and circles.



$$\int_{-3}^{3} f(x)dx = \int_{-3}^{0} x+3\,dx + \int_{0}^{3} \sqrt{9-x^2}\,dx$$

$$= \frac{1}{2}(3)(3) + \frac{\pi(3)^2}{4}$$ Quarter circle.

$$= \boxed{\frac{9}{2} + \frac{9}{4}\pi}$$

Problem 2 .

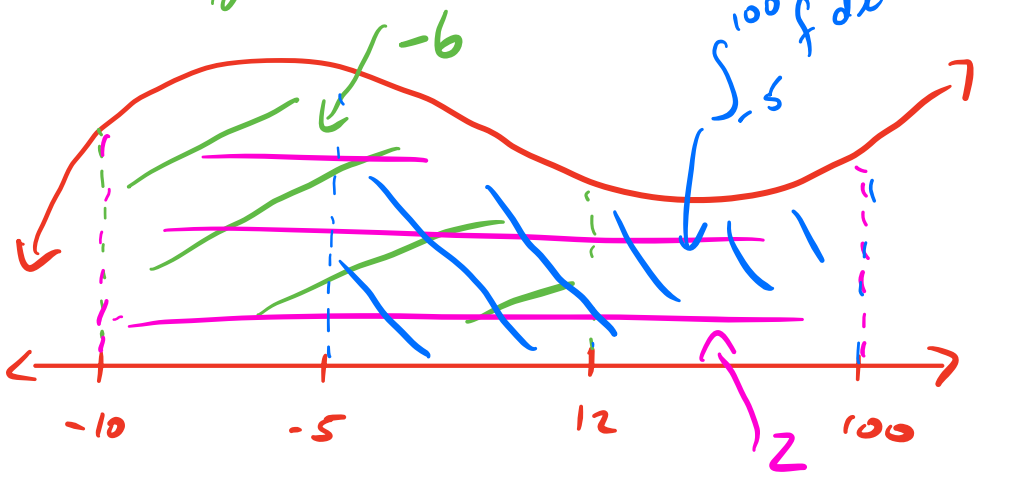
Evaluate the following integral. Given a Riemann integrable function $f : \mathbb{R} \to \mathbb{R}$ and

$$\int_{12}^{-10} f(x)dx = 6, \quad \int_{100}^{-10} f(x)dx = -2, \text{ and } \int_{100}^{-5} f(x)dx = 4.$$

Use properties of the indefinite integral to calculate

$$\int_{-5}^{12} f(x)dx.$$

$6 = \int_{12}^{-10} f\,dx = -\int_{-10}^{12} f(x)\,dx$



$0 = \int_{12}^{100} f\,dx$

$-2 = \int_{100}^{-10} f(x)\,dx$

$= -\int_{-10}^{100} f(x)\,dx$

$= \int_{-10}^{100} f\,dx - \int_{-10}^{12} f\,dx$

$= 2 - (-6) = 8$

$\int_{-5}^{12} f\,dx = \int_{-5}^{100} f\,dx - \int_{12}^{100} f\,dx = -4 - 8 = \boxed{-12}$