Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 1, January 16, 2024

Problem 1 .
Given the function

$$g(x) = \sqrt{5x-3}.$$

(1 point)

1. What is the domain and range of g?

well we need $5x-3 \geq 0 \Rightarrow x \geq \frac{3}{5}$.

Therefore the domain of g is

$[3/5, \infty)$

(4 points)

2. Find the inverse g^{-1}.

We know $g(x)=y \Leftrightarrow g^{-1}(y)=x$.

Then $\sqrt{5x-3} = y$ (1 point)

$\Rightarrow 5x-3 = y^2$ (1 point)

$\Rightarrow 5x = y^2+3$

$\Rightarrow x = \frac{y^2+3}{5}$. (1 point)

So $\boxed{g^{-1}(x) = \frac{y^2+3}{5}}$ (1 point)

* Test: $\sqrt{5\left(\frac{y^2+3}{5}\right)-3} = \sqrt{y^2+3-3} = y$, $y \geq 0$.

(5 points)

Problem 2 .

Given two functions

$$f = \ln(x+1) \text{ and } g = \frac{x^2-5x+5}{x-2}$$

1. Find the domain of $f \circ g(x) = f(g(x))$ and write it in interval notation.

well $f\circ g(x) = \ln(g(x)+1)$

$$= \ln\left(\frac{x^2-5x+5}{x-2} + 1\right)$$ (1 point)

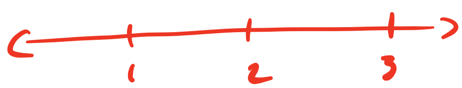
For $f\circ g$ to be defined we need

$$\frac{x^2-5x+5}{x-2} + 1 > 0$$ (1 point)

Then $$\frac{x^2-5x+5}{x-2} + 1 = \frac{x^2-5x+5}{x-2} + \frac{x-2}{x-2}$$

$$= \frac{x^2-4x+3}{x-2}$$ (1 point)

$$= \frac{(x-3)(x-1)}{(x-2)}$$ (>0 is what we need)



	$(-\infty,1)$	$(1,2)$	$(2,3)$	$(3,\infty)$
$x-3$	$-$	$-$	$-$	$+$
$x-1$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$+$	$+$
	$-$	$+$	$-$	$+$

(1 point)

(1 point)

So $\frac{(x-3)(x-1)}{(x-2)} > 0$ on $\boxed{(1,2) \cup (3,\infty)}$ The domain of $f\circ g$.