Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 7, October 19, 2023

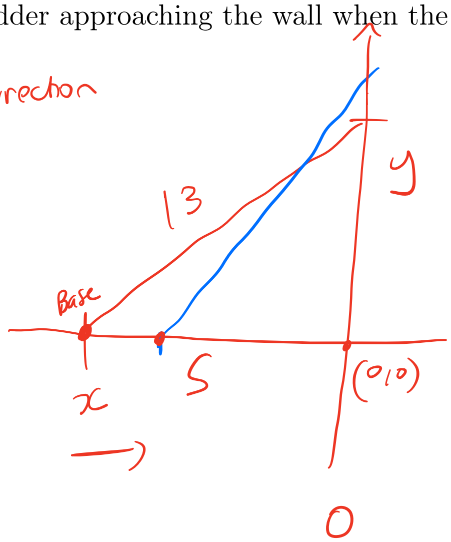
(6 points)

(-0.5 if sign mistake)

Problem 1 A ladder 13 meters long rests on horizontal ground and leans against a vertical wall. The top of the ladder is being pulled up the wall at 0.1 meters per second. How fast is the foot of the ladder approaching the wall when the foot of the ladder is 5 m from the wall?

→ + direction

Change in S is negative but change in x is positive.



constant $\frac{dy}{dt} = 0.1 \ \frac{m}{sec}$

$S = -x$

$\frac{dS}{dt} = \frac{-dx}{dt}$

Find $\left.\frac{dS}{dt}\right|_{S=5}$

(2 points) $S^2 + y^2 = 13$

$\frac{d}{dt}(S^2 + y^2) = \frac{d}{dt}(13)$ (1 point)

$2S\frac{dS}{dt} + 2y\frac{dy}{dt} = 0$

$\sqrt{13^2 - 5^2}$
$= \sqrt{169 - 25}$
$= \sqrt{144}$
$= 12$

(1 point) $\frac{dS}{dt} = \frac{-2y \cdot \frac{dy}{dt}}{2S} = \frac{-y\frac{dy}{dt}}{S}$

(1 point)

$\left.\frac{dS}{dt}\right|_{S=5} = \frac{-12 \cdot 0.1}{5} \Rightarrow \left.\frac{dS}{dt}\right|_{S=5} = \frac{-12}{50}$ (1 point)

(1 point)

$\Rightarrow \left.\frac{dx}{dt}\right|_{S=5} = -\frac{dS}{dt} = \boxed{\frac{12}{50}} \ \frac{ft}{sec}$

Problem 2 .

2 points

a.) Use linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 8$ to find an approximate value for $f(7.76)$.

A.) 1.975

B.) 1.98

C.) 1.985

D.) 1.99

E.) 1.995

$L(x) = f'(a)(x-a) + f(a)$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f'(8) = \frac{1}{3} \cdot \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{12}$

$f(8) = 2$

$L(x) = \frac{1}{12}(x-8) + 2$

$L(7.76) = \frac{1}{12}(7.76-8) + 2 = \frac{1}{12}(-0.24) + 2$

$= \boxed{1.98}$

$\frac{-24}{1200} + 2$

$= \frac{-1}{50} + 2$

$= \frac{-2}{100} + \frac{200}{100}$

$= \frac{198}{100}$

$$\begin{array}{r} 50 \\ 24 \overline{)1200} \\ 120 \\ \hline 0 \\ 0 \\ \hline 0 \end{array}$$

−0.5 if correct solution, but couldn't simplify.

2 points

b.) Find an approximate value for $\ln(2.6)$ using linear approximation. (Hint $e \approx 2.718281...$.)

Denote $f(x) = \ln(x)$

Then $f'(x) = \frac{1}{x}$

$L(x) = f'(a) \cdot (x-a) + f(a)$. Choose $a = e$.

Then $f'(e) = \frac{1}{e}$, $f(e) = \ln(e) = 1$

So $L(x) = \frac{1}{e} \cdot (x - e) + 1$

Then $f(2.6) \approx L(2.6) = \frac{1}{e}(2.6 - e) + 1$