Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 9, November 2, 2023

Problem 1 Given

$$f(x) = \frac{x^2}{x^2+3}$$

(3 points)

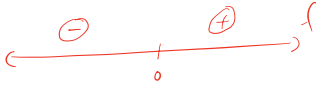
a.) Find all the intervals on which f is increasing and decreasing:

$$f'(x) = \frac{2x\cdot(x^2+3) - x^2(2x)}{(x^2+3)^2}$$

$$= \frac{2x^3+6x-2x^3}{(x^2+3)^2}$$

$$= \frac{6x}{(x^2+3)^2} \quad (1 \text{ point})$$

f' can only change sign at $x=0$



- Since $f'(x) > 0$ for all $x \in (0,\infty)$ f is increasing on $(0,\infty)$ (1 point)
- Since $f'(x) < 0$ for all $x \in (-\infty,0)$ f is decreasing on $(-\infty,0)$ (1 point)

(1 point)

b.) Find the local minima and maxima values of f:

The local extrema is at $x=0$ and

from a) we see f has a local minimum at 0.

(3 points)

c.) Find the intervals of concavity and inflection points.

$$f''(x) = \frac{d}{dx}\left(6x(x^2+3)^{-2}\right)$$

$$= 6(x^2+3)^{-2} + 6x(-2(x^2+3)^{-3}\cdot 2x)$$

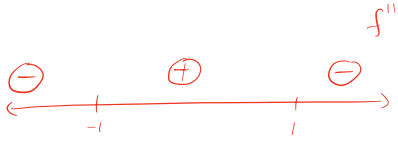
$$= \frac{6(x^2+3)}{(x^2+3)^3} - \frac{24x^2}{(x^2+3)^3}$$

$$= \frac{6x^2+18-24x^2}{(x^2+3)^3}$$

$$= \frac{-18x^2+18}{(x^2+3)^3}$$

$$= \frac{-18(x+1)(x-1)}{(x^2+3)^3}$$

(1 point)



So f concave up on $(-1,1)$ (1 point)

And f concave down on $(-\infty,-1)\cup(1,\infty)$ (1 point)

(3 points)

Problem 2 .

Which of the followin value(s) of x satisfy the Mean Value Theorem for

$$f(x) = x^3 - 2x^2 - 4x + 1$$

on the interval $[0,2]$?

A.) $x = -\frac{4}{3}, \frac{4}{3}$

B.) $x = 2$

C.) $x = 0$ only.

D.) $x = \frac{4}{3}$ only.

E.) $x = 0, \frac{4}{3}$

MVT:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Find c that works

$f'(x) = 3x^2 - 4x - 4$ (1 point)

$f(a) = f(0) = 1$

$f(b) = f(2) = 2^3 - 2 \cdot 2^2 - 4 \cdot 2 + 1 = -7$

So $\frac{f(2) - f(0)}{2} = \frac{-7 - 1}{2} = -4$ (1 point)

And fun solve $3x^2 - 4x - 4 = -4$

$\Rightarrow 3x^2 - 4x = 0$

$\Rightarrow (x)(3x - 4) = 0$

$\Rightarrow x = 0$ or $x = \frac{4}{3}$

But 0 not in $(0,2)$ so it cannot satisfy MVT. Only $x = \frac{4}{3}$. (1 point)