Name: Solutions

MAC 2311 - Analytical Geometry and Calculus I
Quiz # 9, March 19, 2024

(5 points)

Solve $f'(x) > 0$

Problem 1 Find the intervals where

$$f(x) = x^3 - 12x + 1$$

is strictly increasing .

a.) $(-\infty, -2)$

b.) $(-2, \infty)$

c.) $(-2, 2) \cup (2, \infty)$

d.) $(-2, 2)$

e.) $\cancel{(-\infty, 2) \cup (2, \infty)}$

$(-\infty, -2) \cup (2, \infty)$

Step 1: $f'(x) = 3x^2 - 12$

Step 2: Solve $f'(x) > 0$

$\Rightarrow \quad 3x^2 - 12 > 0$

$\Rightarrow \quad 3(x-2)(x+2) > 0$

	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$x-2$	$-$	$+$	$+$
$x+2$	$-$	$-$	$+$
$(x-2)(x+2)$	$+$	$-$	$+$

f is strictly increasing on

$(-\infty, -2) \cup (2, \infty)$

Problem 2 .

Suppose f is a function defined on the closed and bounded interval $[2,5]$,

- f is continuous on $[2,5]$,
- f differentiable on $(2,5)$,
- $f(2) = -2$ and $f'(x) \geq 1$ for all x in $[2,5]$.

(1 point)

a.) Write down the conclusion of the Mean Value Theorem using the function f and interval [2,5] ~~[2,4]~~ above. Fill in as much information as possible.

$$\frac{f(b)-f(a)}{b-a} = f'(c) \quad \text{(conclusion of MVT)}$$

we know

since $f'(x) \geq 1$

$$\frac{f(5)-f(2)}{5-2} = f'(c) \geq 1$$

(4 points)

b.) How small can $f(5)$ be? That is what is the best lower bound on $f(5)$? (Hint: Use the Mean Value theorem and your work from a.))

we know $$\frac{f(5)+2}{3} = f'(c) \geq 1$$

$$\Rightarrow \frac{f(5)+2}{3} \geq 1$$

$$\Rightarrow f(5)+2 \geq 3$$

$$\Rightarrow f(5) \geq 1$$

That is the smallest $f(5)$ can be is $\underline{1}$.