**Name:** Solutions

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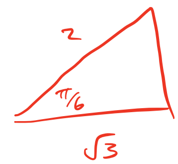
**MAC 2311 - Analytical Geometry and Calculus I**

Quiz # 8, March 5, 2024




(2 points)

**Problem 1** Given $y = t + sin(t) + 2$ as $t$ goes from $2\pi$ to $\frac{13\pi}{6}$.

a.) Calculate $\Delta y$: let $f(t) = t + \sin(t) + 2$.

$t_0 = 2\pi$, $t_1 = \frac{13\pi}{6}$

$$\Delta y = f(t_1) - f(t_0)$$
$$= f\left(\tfrac{13\pi}{6}\right) - f(2\pi)$$
$$= \left(\tfrac{13\pi}{6} + \tfrac{1}{2} + 2\right) - (2\pi + 0 + 2)$$
$$= \tfrac{\pi}{6} + \tfrac{1}{2}$$
$$= \boxed{\tfrac{\pi}{6} + \tfrac{1}{2}}$$

$$\sin\left(\tfrac{13\pi}{6}\right) = \sin\left(\tfrac{\pi}{6}\right) = \tfrac{1}{2}$$

(2 points)

b.) Calculate $dy$:

$$dy = f'(t)\,dt$$
$$= (\cos(t) + 1)\,dt$$
$$\approx (\cos(t_0) + 1)\,\Delta t$$
$$= (\cos(2\pi) + 1) \cdot \tfrac{\pi}{6}$$
$$= \boxed{\tfrac{\pi}{3}}$$

, $\cos(2\pi) = 1$

$$\Delta t = t_1 - t_0 = \tfrac{13\pi}{6} - \tfrac{2\pi}{6} = \tfrac{\pi}{6}$$

**Problem 2** .

(3 points)

a.) Use linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 8$ to find an approximate value for $f(7.76)$.

A.) 1.975

B.) 1.98

(1)

C.) 1.985

D.) 1.99

E.) 1.995

- $f(x) = x^{\frac{1}{3}}$, $f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$ (1 point)
- $L_a(x) = f'(a)(x-a) + f(a)$, $a = 8$
- $f'(a) = \frac{1}{3} \cdot \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$ (1)
- $f(a) = \sqrt[3]{8} = 2$

$f(7.76) \approx 1.98$

$\Rightarrow L_a(x) = \frac{1}{12}(x-8) + 2$ (1)

$f(7.76) \approx L_a(7.76) = \frac{1}{12} \cdot (7.76 - 8) + 2$
$= \frac{1}{12}\left(-\frac{24}{100}\right) + 2 = \frac{-2}{100} + \frac{200}{100} = \frac{198}{100} = \boxed{1.98}$

(3 points)

b.) Find an approximate value for $\ln(2.6)$ using linear approximation.(Hint $e \approx 2.718281...$ .)

A.) $\frac{1}{e}(2.6 - e) + 1$

B.) $\frac{1}{e}(2.6 - 2e) + 1$

C.) $\frac{1}{e}(2.6 + e) + 1$

D.) $\frac{-1}{e}(2.6 - e) + 1$

E.) $\frac{1}{e}(2.6 - e) + 1$

$f(x) = \ln(x)$, $f(e) = \ln(e) = 1$

$f'(x) = \frac{1}{x}$, $f'(e) = \frac{1}{e}$

$a = e$

$L_e(x) = f'(e)(x-e) + f(e)$
$= \frac{1}{e}(x - e) + 1$ (1 point)

$\ln(2.6) = f(2.6)$
$\approx L_e(2.6)$
$= \frac{1}{e}(2.6 - e) + 1$ (1 point)