Discussion Point 1

MAC 2313

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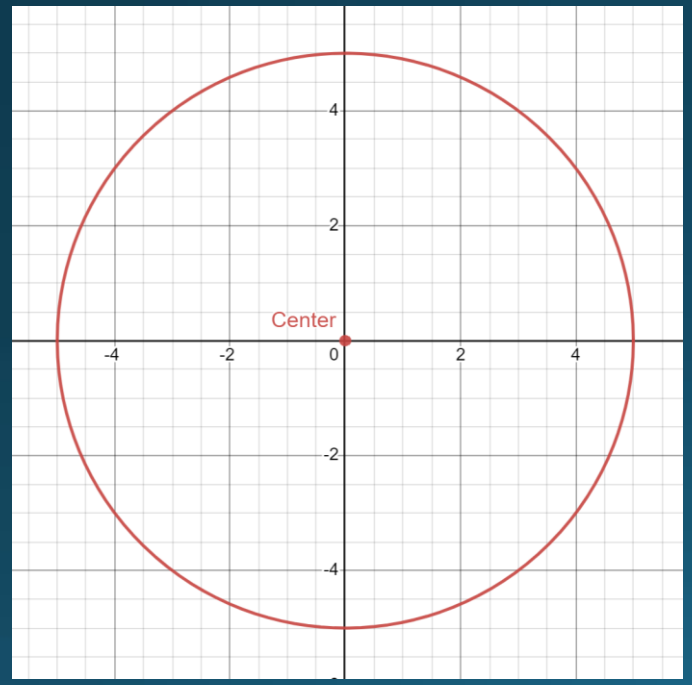
Background

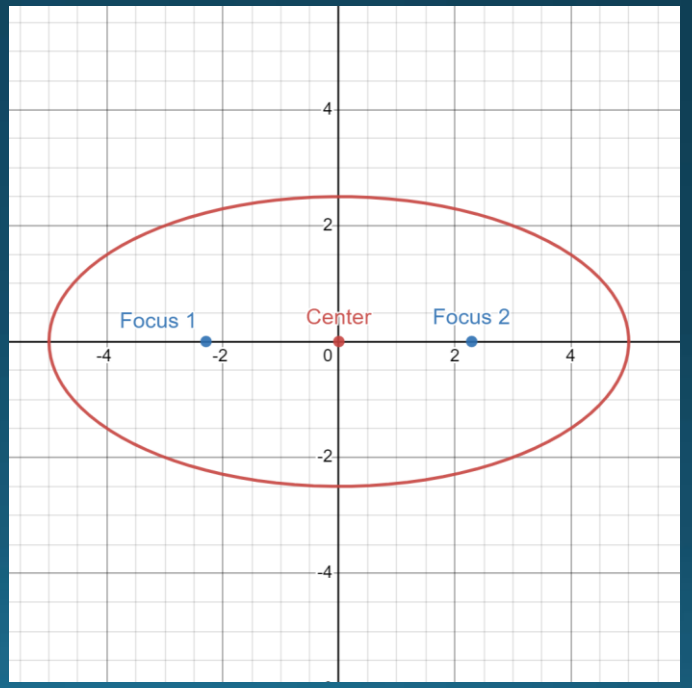
2D conics can be described with either an equation or as through the use focus points.

Circle



Fixed distance from "center"

Ellipse

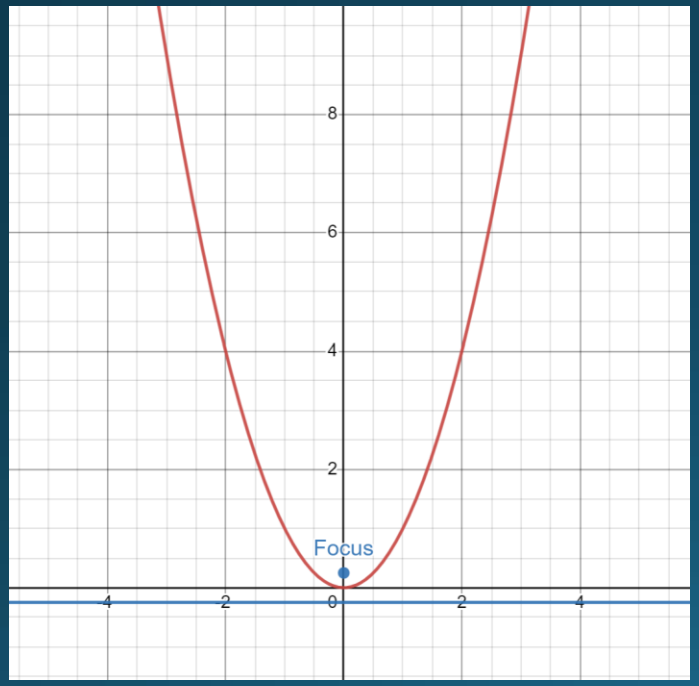


Fixed (modified) distance from "center"
OR
Fixed sum of distance from "foci" points

Background

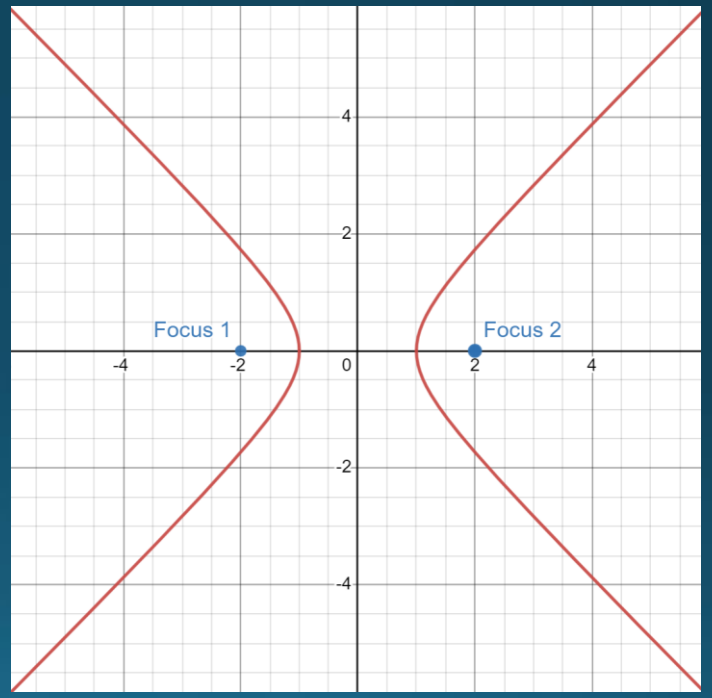
2D conics can be described with either an equation or as through the use focus points.

Parabola



Equidistant from "focus" and line

Hyperbola



Absolute difference between distances from "foci" points

OR

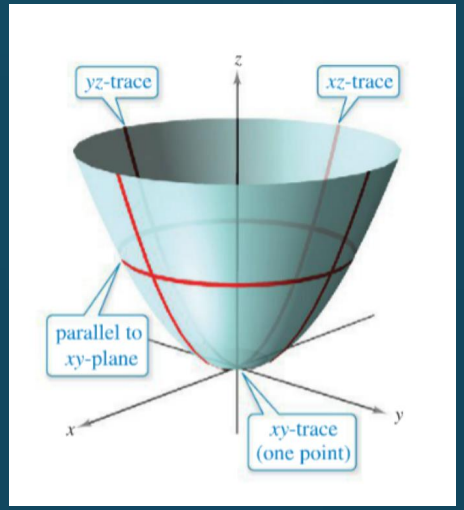
Difference between distances from horizontal/vertical lines (x-,y-axis)

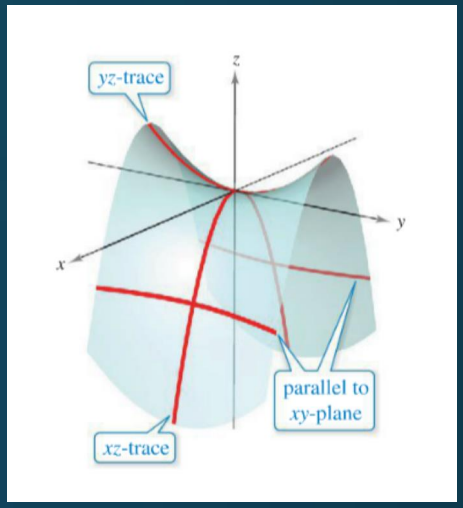
Problem

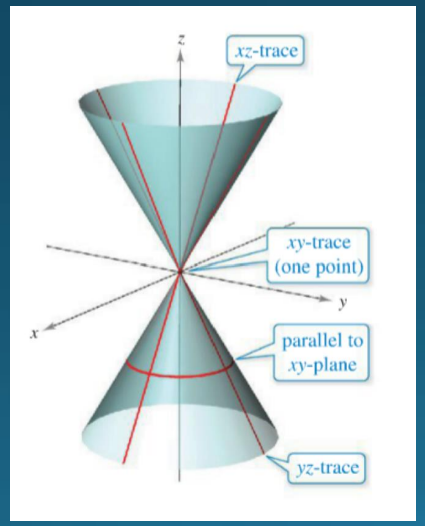
How can we describe the quadric surfaces in a similar way?

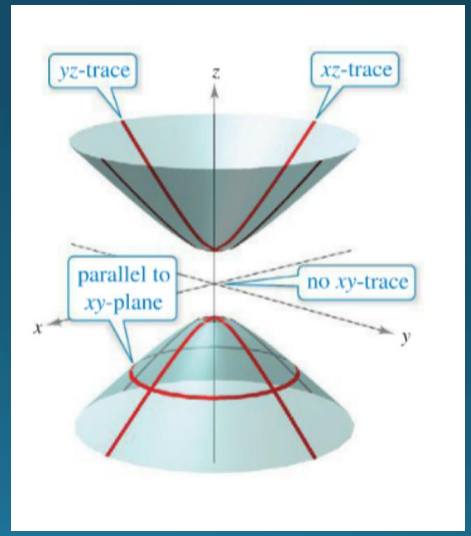
Hints

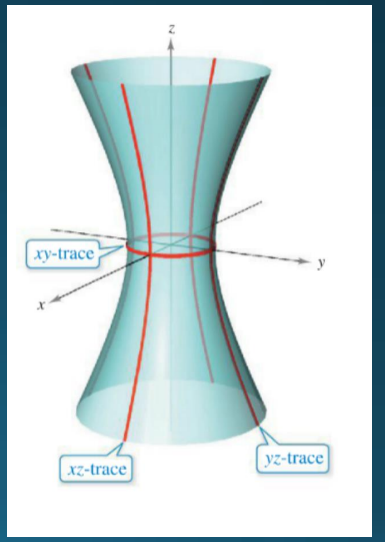
1. Look at the traces. Where are the focus points/lines located on the trace?
2. All of following are sets of points (sometimes nearly) equidistant from the foci objects











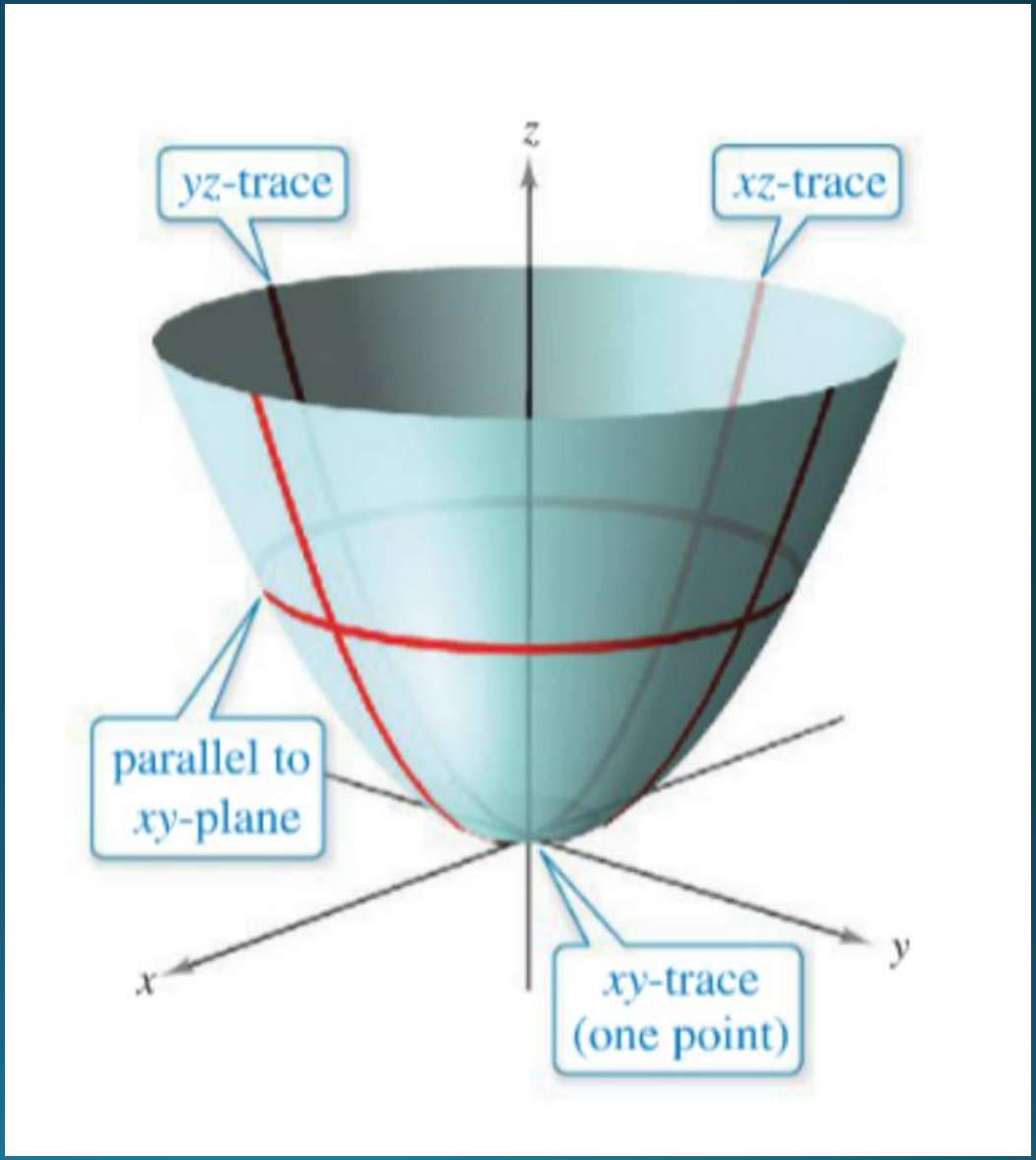
Elliptical Paraboloid

The xz- and yz-traces are both parabolas, suggesting that there are two focus objects: top and bottom.

The top focus object looks like a point in both traces. So, it is most likely a point.

The bottom focus object looks like a line in the x-direction (xz trace) and a line in the y-direction (yz trace). The object is most likely a plane.

Therefore, the elliptical paraboloid is the set of points equidistant to a point and a plane.



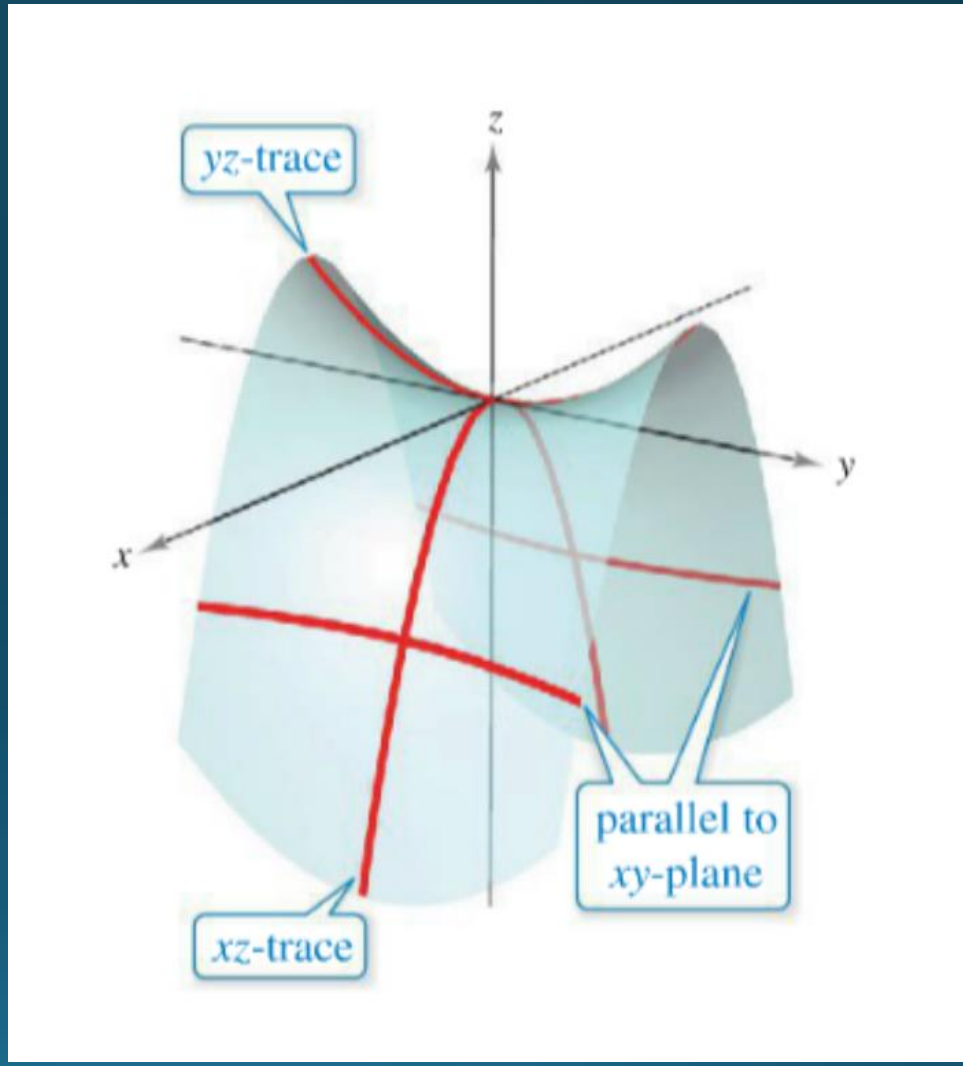
Elliptical Paraboloid

The xz- and yz-traces are both parabolas, suggesting that there are two focus objects: top and bottom.

The top focus object looks like a point in the yz-trace and a line in the xz-trace. So, it is most likely a line.

The bottom focus object looks like a line in the yz-trace and a point in the xz-trace. So, it is most likely a line.

Therefore, the elliptical paraboloid is the set of points equidistant to two perpendicular skew lines.



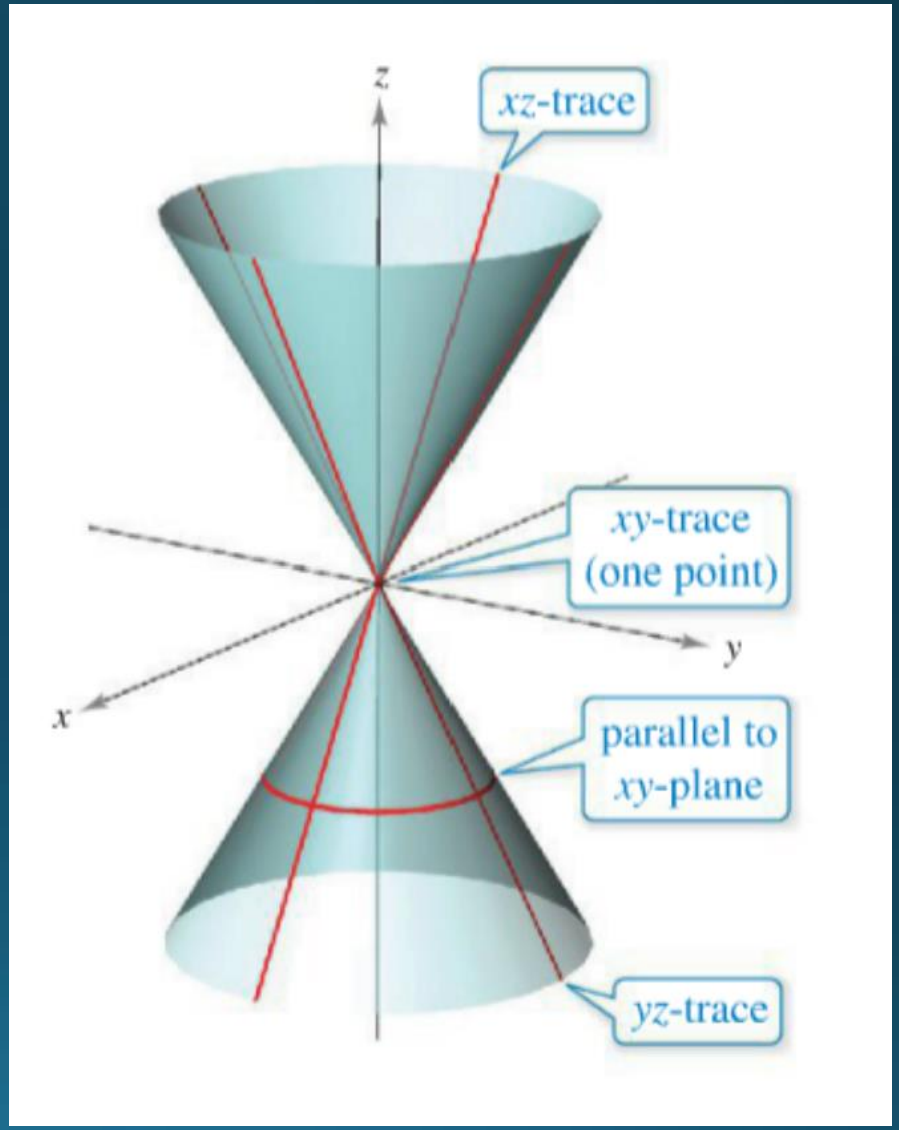
(Elliptical) Cone

The xz- and yz-traces are both hyperbolas. We will use the "line" characteristic of the hyperbola.

The vertical focus object simulates the z-axis in the xz-trace and the yz-trace. However, it appears as a point in the xy-traces (circle traces). The object is a vertical line.

The horizontal focus object looks like a line in both the yz-trace and the xz-trace. So, it is most likely a horizontal plane.

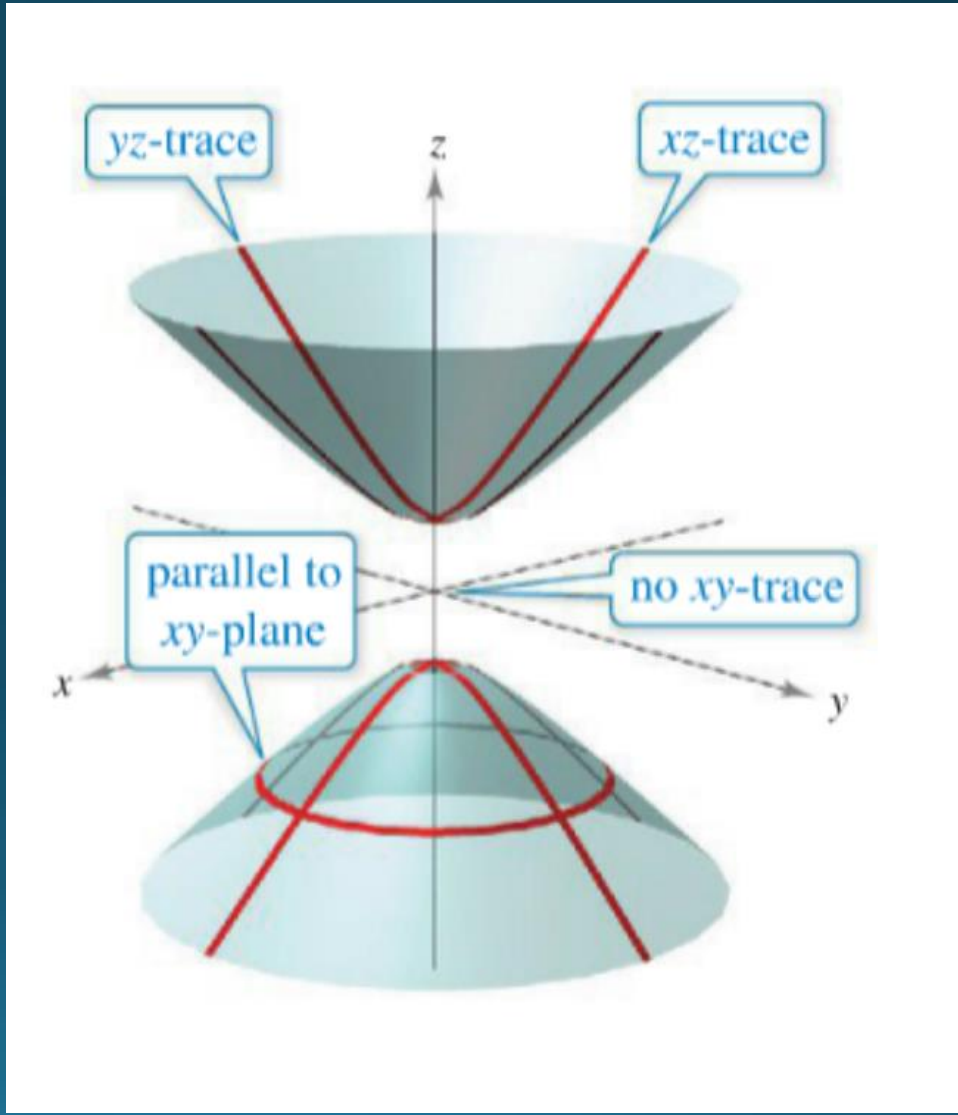
Therefore, the cone is the set of points equidistant to a plane and an orthogonal line.



Hyperboloid (2 sheet)

Similar to the cone analysis except 1 key difference: the set of points seems to be closer to the vertical line than the horizontal plane.

The hyperboloid of 2 sheets is the set of points such that the points are k (some constant) units closer to the normal line than the plane.



Hyperboloid (2 sheet)

Vice versa for the 1 sheet hyperboloid.

The hyperboloid of 2 sheets is the set of points such that the points are k (some constant) units closer to the plane than the normal line.