[2 points] 1. Let $\vec{u} = \langle 1, 0, -1\rangle$ and $\vec{v} = \langle 8, -4, -1\rangle$.
(a). Find ANY vector that is orthogonal to both u and v. (Do not give me the zero vector).

Solution:
The cross product of two vectors is a vector that is orthogonal to both. So, simply find $\vec{u} \times \vec{v}$.

$$\begin{bmatrix} i & j & k \\ 1 & 0 & -1 \\ 8 & -4 & -1 \end{bmatrix} \implies \vec{u} \times \vec{v} = \langle 0-4, -(-1+8), -4\rangle = \boxed{\langle -4, -7, -4\rangle}.$$

Note: any multiple of $\langle -4, -7, -4\rangle$ will work.

[3 points] (b). Find the angle between u and v.

Solution:
The standard way is using the equation

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos(\theta).$$

So,

$$\begin{aligned} \vec{u} \cdot \vec{v} &= 1(8) + 0(-4) + (-1)(-1) = 9. \\ |\vec{u}| &= \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}. \\ |\vec{v}| &= \sqrt{8^2 + (-4)^2 + (-1)^2} = \sqrt{81} = 9. \end{aligned}$$

Putting these facts together,

$$\begin{aligned} 9 &= \sqrt{29}\cos(\theta) \\ \frac{1}{\sqrt{2}} &= \cos(\theta) \end{aligned}$$

$$\boxed{\theta = \frac{\pi}{4}}$$

Note: a similar solution can be made using $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin(\theta)$; however, this method cannot tell the difference between $\theta = \pi/4$ and $\theta = 3\pi/4$.

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2. [3 points] Find the vector equation for the line that passes through the point $(2, 0, 1)$ and the center of the sphere $(x+2)^2+(y-2)^2+(z-5)^2 = 36$.

Solution:
The center of the sphere is $(-2, 2, 5)$. To make the equation for the line, a starting point and a direction are needed.
Possible starting points: $(2, 0, 1)$ or $(-2, 2, 5)$ (or maybe the midpoint).
Possible directions: vector from $(2, 0, 1)$ to $(-2, 2, 5)$ (or some multiple of that vector).

The vector from $(2, 0, 1)$ to $(-2, 2, 5)$ is $\langle -4, 2, 4\rangle$. Using $(2, 0, 1)$ as the starting point, we get

$$L(t) = \boxed{\langle 2-4t, 2t, 1+4t\rangle}.$$