1. A quadric surface is given by the equation $-x^2+y^2+6y+z^2-10z+30=0$.
(a) [~~1~~ 2 points] Simplify the equation via completing the squares.
Solution:

$$\begin{aligned} -x^2+y^2+6y+z^2-10z+30&=0 \\ -x^2+y^2+6y+9-9+z^2-10z+25-25+30&=0 \\ -x^2+(y+3)^2+(z-5)^2-4&=0 \end{aligned}$$

$$\boxed{-x^2+(y+3)^2+(z-5)^2=4}$$

(b) [~~3~~ 2 points] Identify the trace of the surface when intersected by the planes $x=k$. (Note: if the trace is different for different values of k, name each trace and the corresponding k-values where they apply).
Solution: Set $x=k$. We then have

$$\begin{aligned} -k^2+(y+3)^2+(z-5)^2&=4 \\ (y+3)^2+(z-5)^2&=4+k^2 \end{aligned}$$

Since $4+k^2$ is always a positive number, the only shape the trace can be is a $\boxed{\text{circle}}$ (If a the right-side was 0, then we would have a point. If the right-side was negative, the trace would be nothing).

(c) [1 point] Identify the surface.

Solution: The surface seems to be made up of circles. The circles shrink as one gets closer to $x=0$. The only quadric surface that does that is the $\boxed{\text{hyperboloid on one sheet}}$.

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2. Let $\vec{r}(t) = \langle 3t,\ 1+t^2,\ 2t\ln(t-1)\rangle$.
(a) [1.5 points] Find $\vec{r}\,'(t)$.
Solution: You may take the derivative component-wise.

$$\boxed{\langle 3,\ 2t,\ 2\ln(t-1) + 2\frac{t}{t-1}\rangle}$$

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(b) [1.5 points] Find $\hat{T}(2)$, the *unit* tangent vector to the curve $\vec{r}(t)$ at $t = 2$.

Solution: Since we have completed all differentiation steps, we can plug in $t = 2$ and then calculate $\hat{T}(2)$ (the reverse is possible but more annoying to perform).

$$\begin{aligned} \vec{r}(2) &= \langle 3, 4, 2\ln(1) + 2\rangle = \langle 3, 4, 4\rangle \\ |\vec{r}(2)| &= \sqrt{3^2 + 4^2 + 4^2} = \sqrt{41} \\ \hat{T}(2) &= \frac{\vec{r}(2)}{|\vec{r}(2)|} = \boxed{\frac{1}{\sqrt{41}}\langle 3, 4, 4\rangle}. \end{aligned}$$