1. [2 points] Evaluate the integral.

$$\int_0^2 \int_0^5 10xe^{xy}\, dydx$$

Solution:

$$\begin{aligned}
\int_0^2 \int_0^5 10xe^{xy}\, dydx &= \int_0^2 10x\frac{e^{xy}}{x}\bigg|_0^5\, dx \\
&= \int_0^2 10e^{5x} - 10\, dx \\
&= 2e^{5x} - 10x\bigg|_0^2 \\
&= (2e^{10} - 2) - 10(2) \\
&= \boxed{2e^{10} - 22}.
\end{aligned}$$

2.[2 points] Swap the order of integration. DO NOT SOLVE THE INTEGRAL.

$$\int_0^1 \int_{x^2}^1 \frac{1}{2y}\, dydx$$

Solution: See picture below (credits to Desmos).

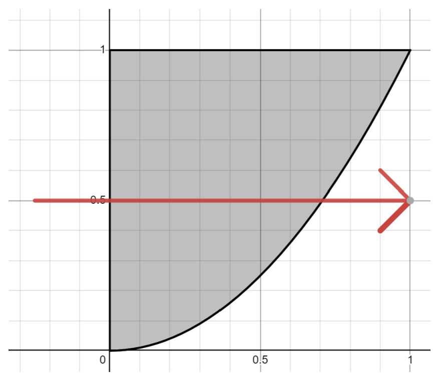


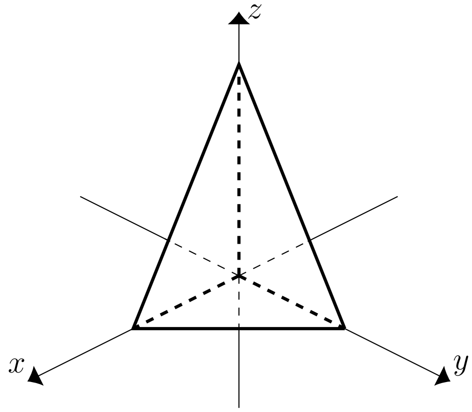
Figure 1: Region of integration. The arrow is used to determine upper and lower bounds for dx

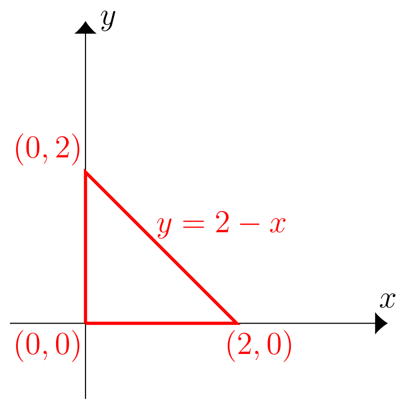
Based on the figure in the graph, the first border the arrow crosses is the $x = 0$ line. Afterwards, it crosses the $x = \sqrt{y}$ curve. This means the x-bounds will be from 0 to $\sqrt{y}$. For the y-bounds, we no longer have access to x (as it got integrated out), so we only care about the min/max that y can be. Thus, the y-bounds will be from 0 to 1, so

$$\boxed{\int_0^1 \int_0^{\sqrt{y}} \frac{1}{2y}\, dxdy}$$

3. [4 points] Find the volume under the plane $z = 4 - 2x - 2y$ in the first octant through the following steps.

(a) The left picture is an (unmarked) isometric view of the object. Sketch the object in top-down view on the right. The coordinate axes are labelled to help orientation. Mark anything that might be relevant.





(b) Set up the integral for the volume. The shape of your answer in (a) should be a guide to the correct integration bounds.

$$\int_0^2 \int_0^{2-x} 4 - 2x - 2y \; dy dx$$

(c) Evaluate the above integral.

Solution

$$\begin{aligned}
\int_0^2 \int_0^{2-x} 4 - 2x - 2y \; dydx &= \int_0^2 4y - 2xy - y^2 \Big|_0^{2-x} \; dx \\
&= \int_0^2 4(2-x) - 2x(2-x) - (2-x)^2 \; dx \\
&= \int_0^2 (4-2x)(2-x) - (2-x)^2 \; dx \\
&= \int_0^2 2(2-x)^2 - (2-x)^2 \; dx \\
&= \int_0^2 (2-x)^2 \; dx \\
&= \frac{-1}{3}(2-x)^3 \Big|_0^2 \\
&= 0 + \frac{8}{3} = \boxed{\frac{8}{3}}.
\end{aligned}$$