1(a). [2 points] Convert the point $(-\sqrt{2}, \sqrt{2}, 2\sqrt{3})$ from *rectangular* coordinates to *cylindrical* coordinates (r, θ, z).
Solution:

$$z = z = 2\sqrt{3}.$$
$$r^2 = x^2 + y^2 = 2 + 2 = 4 \quad \Rightarrow r = 2$$
$$\tan(\theta) = \frac{y}{x} = -1 \quad \Rightarrow \quad \theta = \frac{3\pi}{4}, \cancel{\frac{7\pi}{4}} \quad \text{(Quadrant II, not IV)}$$

The point in spherical coordinates is $\boxed{\left(2, \frac{3\pi}{4}, 2\sqrt{3}\right)}$.

1(b). [2 points] [2 points] Convert $\left(4, \frac{4\pi}{3}, \frac{\pi}{6}\right)$ from *spherical* coordinates to *rectangular* coordinates (x, y, z).
Solution:
For this problem (and anything involving spherical coordinates), note that $z = \rho\cos(\phi)$, "r" $= \rho\sin(\phi)$, $x =$ "r" $\cos(\theta)$, and $y =$ "r" $\sin(\theta)$. (the variable "r" is not a variable for either rectangular or spherical but it is still a useful quantity when remembering these formulas). Using this, we have

$$x = r\cos(\theta) = \rho\sin(\phi)\cos(\theta) = 4\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)$$
$$= -1$$
$$y = r\sin(\theta) = \rho\sin(\phi)\sin(\theta) = 4\left(\frac{1}{2}\right)\left(\frac{-\sqrt{3}}{2}\right)$$
$$= -\sqrt{3}$$
$$z = \rho\cos(\phi) = 4\left(\frac{\sqrt{3}}{2}\right)$$
$$= 2\sqrt{3}$$

The point in rectangular coordinates is $\boxed{\left(-1, -\sqrt{3}, 2\sqrt{3}\right)}$.

2. [4 points] Calculate the following integral

$$\iint_R 2xy\ dA,$$

where R is the region inside the ellipse $\dfrac{x^2}{9} + y^2 = 1$ *and* the first quadrant. [Hint: perform a change of variables twice (or once if you're clever)].

Solution:

To simplify R, we will perform the change of variables $x = 3u$ and $y = v$. The Jacobian would be

$$\frac{d(x,y)}{d(u,v)} = \begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} = |3| = 3.$$

The first change of variables converts our integration region into a quarter-circle. Our second change of variables will be to polar: $u = r\cos(\theta)$ and $v = r\sin(\theta)$. With this, we obtain:

$$\begin{aligned}
\iint_{\frac{x^2}{9}+y^2\leq 1,\text{QI}} 2xy\ dA &= \iint_{u^2+v^2\leq 1,\text{QI}} 6uv(3\ dA) \qquad \text{(QI = Quadrant I)} \\
&= \int_0^{\pi/2}\int_0^1 18r\cos(\theta)r\sin(\theta)r\ drd\theta \\
&= 9\int_0^{\pi/2} 2\sin(\theta)\cos(\theta)d\theta \int_0^1 r^3\ dr \\
&= 9\sin^2(\theta)\Big|_0^{\pi/2}\left(\frac{r^4}{4}\right)\Big|_0^1 \\
&= 9\,(1)\left(\frac{1}{4}\right) = \boxed{\frac{9}{4}}.
\end{aligned}$$