Fun "proof"

Let $\boxed{a=b} \neq 0$

$(\times a)$ $\quad a^2 = ab$

$(-b^2)$ $\quad a^2 - b^2 = ab - b^2$

$(a+b)\cancel{(a-b)} = b\cancel{(a-b)}$ $\quad / \; (a-b) = 0$

$a+b = b$

$\rightarrow \quad b+b = b$

$2b = b$

$\boxed{a=1}$

HW 19 #1

$(0,\infty)$

$f(x) = x^2 - x - \ln(x)$

$$\boxed{\begin{aligned} &\uparrow : (1,\infty) \\ &\downarrow : (0,1) \end{aligned}}$$

$f'(x) = 2x - 1 - \frac{1}{x}$

$2x - 1 - \frac{1}{x} = 0$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = 1,\ x = \cancel{-\frac{1}{2}}$

$f'(x)$: 0, $\frac{1}{2}$, 1, 2; $-$ at $\frac{1}{2}$, $+$ at 2

$f'(\frac{1}{2}) = \cancel{1-1} - \frac{1}{1/2} = -2 < 0$

$f'(2) = 4 - 1 - \frac{1}{2} = 2.5 > 0$

$$f'(x) = 2x - 1 - \frac{1}{x}$$

$$f''(x) = 2 + \frac{1}{x^2}$$

$$2 + \frac{1}{x^2} = 0$$

$$2x^2 + 1 = 0$$

$$2x^2 = -1$$

$$x^2 = -\frac{1}{2}$$

$f''(x)$: $+$ on $(0, \infty)$; marks at 0 and 1

$\cup$: $(0, \infty)$

$\cap$: never

$$f''(1) = 2 + \frac{1}{1} = 3 > 0$$

* critical points ; the original function must be defined

$$\lim_{x\to 0}\left(\frac{1}{x} - \csc x\right)$$

$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right) \to \frac{1}{0} - \frac{1}{0} \approx \infty - \infty$$

$$\lim_{x\to 0}\left(\frac{\sin x - x}{x\sin x}\right) \overset{LH}{=} \lim_{x\to 0}\left(\frac{\cos x - 1}{x\cos x + \sin x}\right)$$

$$\overset{LH}{=} \lim_{x\to 0}\left(\frac{-\sin x}{\cos x - x\sin x + \cos x}\right) \to \frac{-\sin(0)}{\cos(0) - (0)\sin(0) + \cos(0)}$$

$$\to \frac{0}{1-0+1} = \frac{0}{2} = \boxed{0}$$

$$\lim_{x \to 1^+} (x-1)^{(x^2-1)}$$

$$(x-1) = \left[e^{\ln(x-1)}\right]^{(x^2-1)}$$

$$\lim_{x \to 1^+} e^{\ln(x-1)(x^2-1)}$$

$$= e^{\lim_{x \to 1^+} \underbrace{\ln(x-1)(x^2-1)}} = e^{\lim_{x \to 1^+} \frac{\ln(x-1)}{1/(x^2-1)}}$$

$$e^{\lim_{x\to 1^+} \frac{\ln(x-1)}{1/(x^2-1)}} \overset{LH}{=} e^{\lim_{x\to 1^+} \frac{\frac{1}{x-1}}{\frac{-2x}{(x^2-1)^2}}}$$

$$\frac{d}{dx}\frac{1}{x^2-1} = \frac{(x^2-1)(0)-(1)(2x)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$e^{\lim_{x\to 1^+} \frac{1}{x-1} \cdot \frac{(x^2-1)^2}{-2x}}$$

$$(x^2-1)^2 = (x^2-1)(x^2-1) = (x-1)(x+1)(x-1)(x+1)$$

$$e^{\lim_{x\to 1^+} \frac{1}{\cancel{x-1}} \cdot \frac{(x-1)^{\cancel{2}}(x+1)^2}{-2x}} = e^{\lim_{x\to 1^+} \frac{(x-1)(x+1)^2}{-2x}} \to e^{\frac{(0)(4)}{-2}} = e^0 = \boxed{1}$$