

Fun "proof"

Let $\boxed{a=b} \neq 0 \leftarrow$

($\times a$) $a^2 = ab$

($-b^2$) $a^2 - b^2 = ab - b^2$

$(a+b)(\cancel{a-b}) = b(\cancel{a-b}) \quad / \quad (a-b)$
 $= 0$

$a+b = b$

$\rightarrow b+b = b$

$2b = b$

$\boxed{a=1} \leftarrow$

#W 19 #1

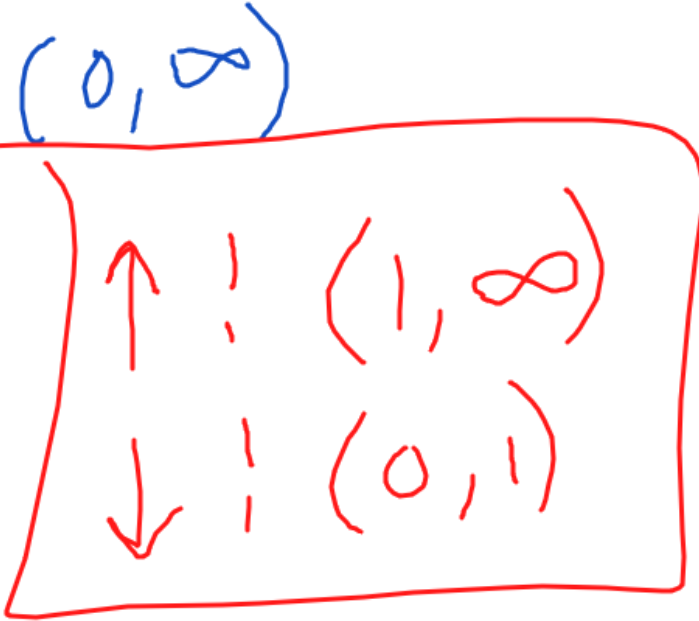
$$f(x) = x^2 - x - \ln(x)$$

$$f'(x) = 2x - 1 - \frac{1}{x}$$
$$2x - 1 - \frac{1}{x} = 0$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$
$$x = 1, x = -\frac{1}{2}$$

$$f'(\frac{1}{2}) = 1 - 1 - \frac{1}{\frac{1}{2}} = -2 < 0$$



$$f'(2) = 4 - 1 - \frac{1}{2} = 2.5 > 0$$

$$f'(x) = 2x - 1 - \frac{1}{x}$$

$$f''(x) = 2 + \frac{1}{x^2}$$

$$2 + \frac{1}{x^2} = 0$$

$$2x^2 + 1 = 0$$

$$2x^2 = -1$$

$$x^2 = -\frac{1}{2}$$

$$f''(1) = 2 + \frac{1}{1} = 3 > 0$$



$(0, \infty)$

never

* critical points: the original function must be defined

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \csc x \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) \rightarrow \frac{1}{0} - \frac{1}{0} \approx \infty - \infty$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin x - x}{x \sin x} \right) \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \left(\frac{\cos x - 1}{x \cos x + \sin x} \right)$$

$$\stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \left(\frac{-\sin x}{\cos x - x \sin x + \cos x} \right) \rightarrow \frac{-\sin(0)}{\cos(0) - (0)\sin(0) + \cos(0)}$$

$$\rightarrow \frac{0}{1-0+1} = \frac{0}{2} = \boxed{0}$$

$$\lim_{x \rightarrow 1^+} (x-1)^{\sqrt{x^2-1}}$$

$$(x-1) = \left[e^{\ln(x-1)} \right]^{\sqrt{x^2-1}}$$

$$\lim_{x \rightarrow 1^+} e^{\ln(x-1) \sqrt{x^2-1}}$$

$$= e^{\lim_{x \rightarrow 1^+} \underbrace{\ln(x-1) \sqrt{x^2-1}}_{\quad}} = e^{\lim_{x \rightarrow 1^+} \frac{\ln(x-1)}{1/\sqrt{x^2-1}}}$$

$$e^{\lim_{x \rightarrow 1^+} \frac{\ln(x-1)}{1/(x^2-1)}} \stackrel{\text{L'H}}{=} e^{\lim_{x \rightarrow 1^+} \frac{\frac{1}{x-1}}{\frac{-2x}{(x^2-1)^2}}}$$

$$e^{\lim_{x \rightarrow 1^+} \frac{1}{x-1} \cdot \frac{(x^2-1)^2}{-2x} \frac{d}{dx} \frac{1}{x^2-1}} = \frac{(x^2-1)(0) - (1)(2x)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$$e^{\lim_{x \rightarrow 1^+} \frac{1}{\cancel{x-1}} \cdot \frac{(x-1)^{\cancel{2}} (x+1)^2}{-2x}} = e^{\lim_{x \rightarrow 1^+} \frac{(x^2-1)(x^2-1)}{-2x}} = e^{\lim_{x \rightarrow 1^+} \frac{(x-1)(x+1)(x-1)(x+1)}{-2x}} \rightarrow e^{\frac{(0)(4)}{-2}} = e^0 = \boxed{1}$$