$$f(x) = \frac{x^2+9}{x}$$

a) find the domain of $f(x) = (-\infty, 0) \cup (0, \infty)$ ←

b) find any asymptotes = VA @ $x=0$

c) find $f'(x)$, $f''(x)$

d) find critical points / Inflection points

e) find intervals of increasing / decreasing

f) " " " concave up / down

g) graph $f(x)$ ←

$$\lim_{x\to\infty} \frac{x^2+9}{x} \xrightarrow{LH} \lim_{x\to\infty} \frac{2x}{1} = 2x \to \infty$$

$$\lim_{x\to-\infty} \frac{x^2+9}{x} \xrightarrow{LH} \lim_{x\to-\infty} 2x \to -\infty$$

<u>no</u> H.A.

$$f(x) = \frac{x^2+9}{x}$$

$$f'(x) = \frac{x(2x)-(x^2+9)(1)}{x^2} = \frac{2x^2-x^2-9}{x^2} = \frac{x^2-9}{x^2}$$

$$f''(x) = \frac{x^2(2x)-(x^2-9)(2x)}{(x^2)^2} = \frac{\cancel{2x^3}-\cancel{2x^3}+18x}{x^4} = \frac{18x}{x^4} = \frac{18}{x^3}$$

$$f'(x) = \frac{x^2-9}{x^2}$$

$f'(x)$ is undefined @ $x=0$ <u>NOT</u> a crit. point

$f'(x)=0 \rightarrow x^2-9=0 \rightarrow (x-3)(x+3)=0$

$\rightarrow x=\pm 3$

$$f(x) = \frac{x^2+9}{x}$$

$$f(3) = \frac{9+9}{3} = \frac{18}{3} = 6$$

$$f(-3) = \frac{9+9}{-3} = -6$$

critical points: $(3,6), (-3,-6)$

$f''(x) = \frac{16}{x^3}$

$f''(x)$ und @ $x=0$ $\rightarrow$ <u>NOT</u> inf. point

$f''(x) = 0$ $\rightarrow$ never

$\boxed{\text{no inflection points}}$

crit. points $(3,6)(-3,-6)$

$$f'(x) = \frac{x^2-9}{x^2}$$

$\rightarrow f'(x)$:

$+$		$-$		$-$		$+$
	-3		0		3	

$$f(-4) = \frac{9}{16}$$

$$f(-1) = \frac{-8}{1}$$

$$f(1) = \frac{-8}{1}$$

$$f(4) = \frac{9}{16}$$

$f(x)$

$\uparrow (-\infty,-3) \cup (3,\infty)$

$\downarrow (-3,0) \cup (0,3)$

$$f''(x) = \frac{18}{x^3}$$

$f''(x) =$

−		+
	0	

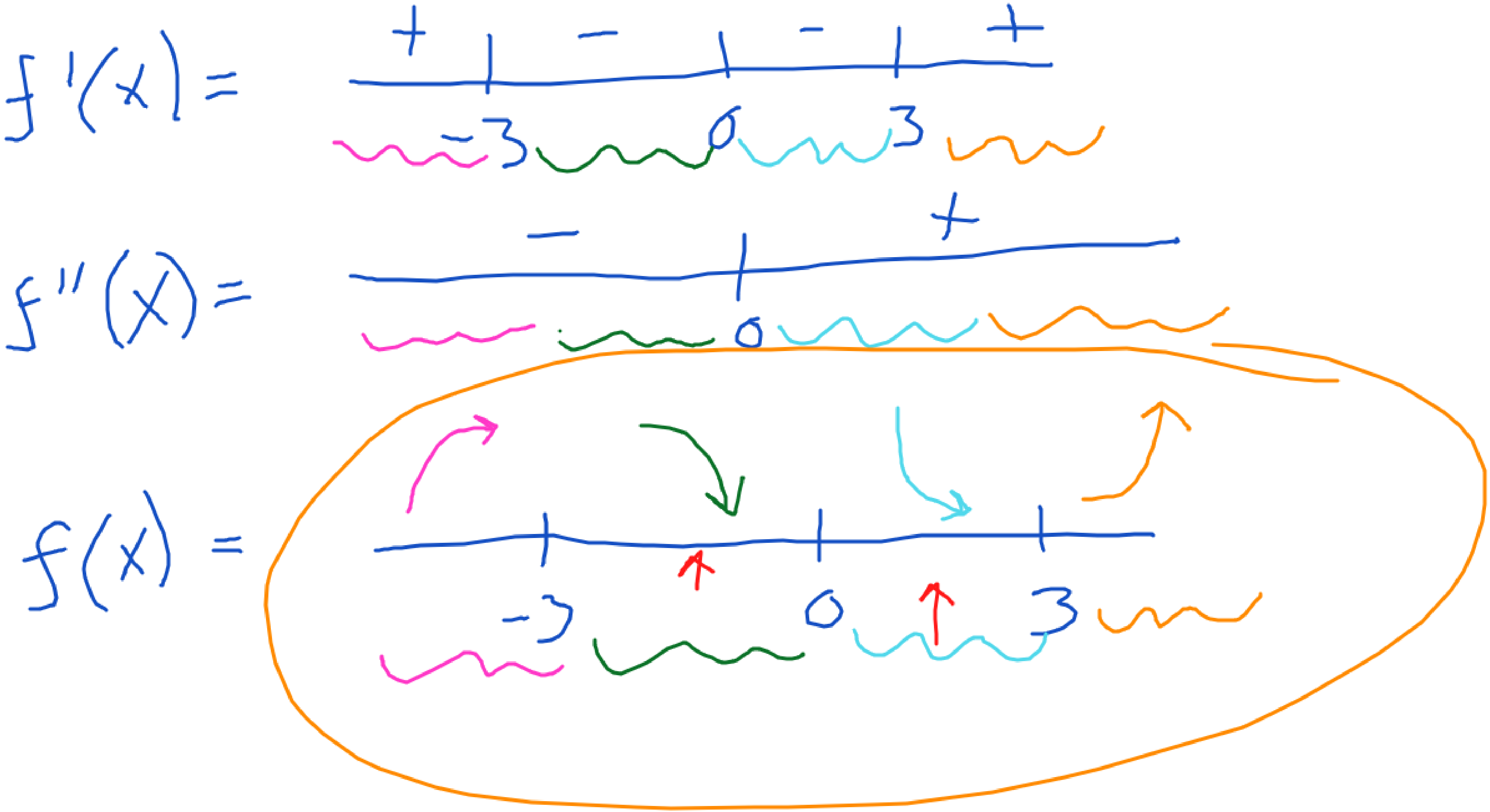
$f(-1) = -18$

$f(1) = 18$

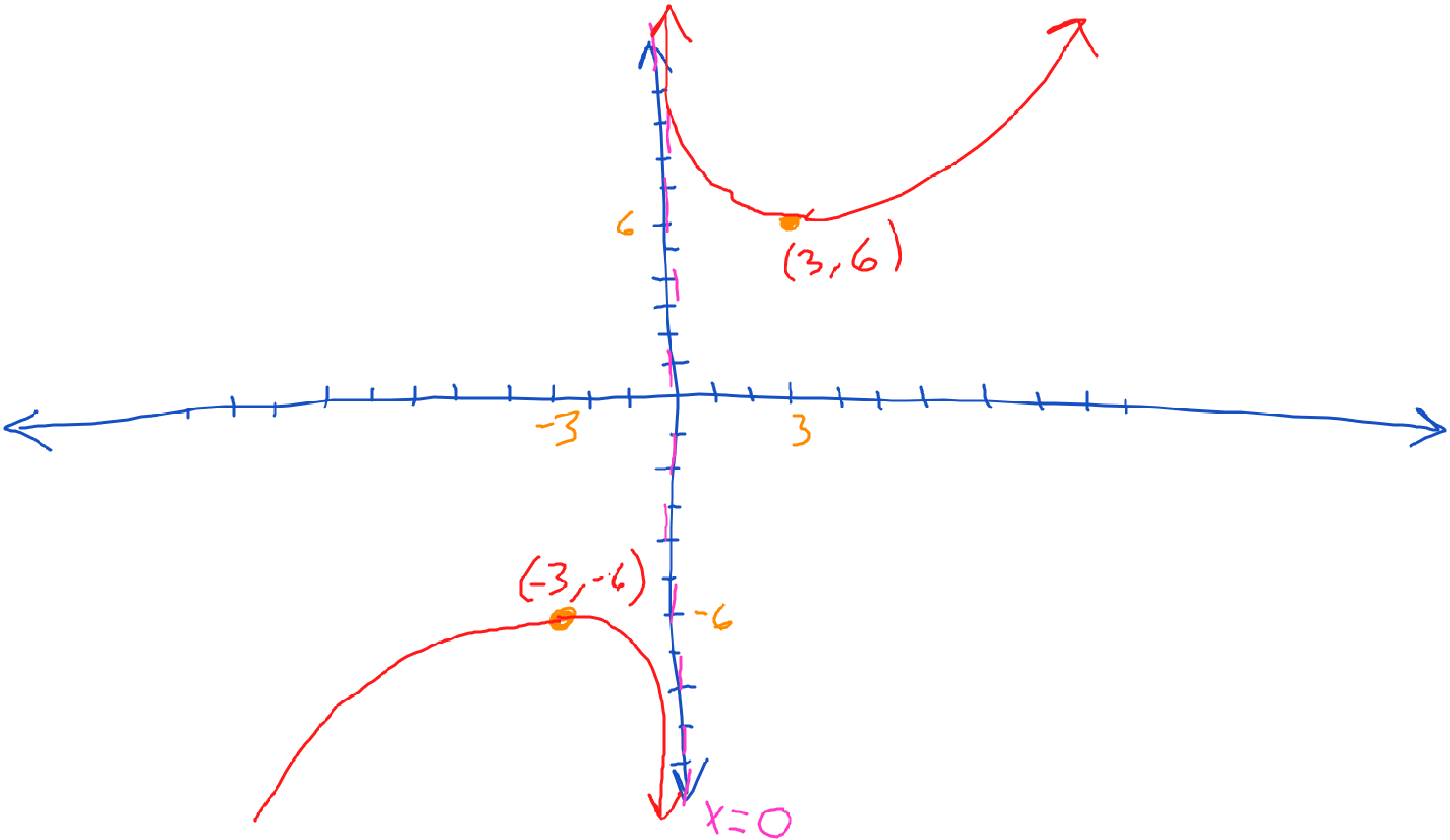
$f(x)$

concave up $(0, \infty)$

" down $(-\infty, 0)$



f'(x) =
+
−
−
+
−3
0
3
f''(x) =
−
+
0
f(x) =
−3
0
3



6
(3, 6)
-3
3
(-3, -6)
-6
x=0