

$$f(x) = \frac{x^2 + 9}{x}$$

- Find the domain of $f(x) = (-\infty, 0) \cup (0, \infty)$ ←
- Find any asymptotes = VA @ $x=0$
- Find $f'(x)$, $f''(x)$
- Find critical points / inflection points
- Find intervals of increasing / decreasing
- " " " " concave up / down
- graph $f(x)$ ←

$$\lim_{x \rightarrow \infty} \frac{x^2 + 9}{x} \xrightarrow{\text{L'H}} \lim_{x \rightarrow \infty} \frac{2x}{1} = 2x \rightarrow \infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 + 9}{x} \xrightarrow{\text{L'H}} \lim_{x \rightarrow -\infty} 2x \rightarrow -\infty$$

} no H.A.

$$f(x) = \frac{x^2 + 9}{x}$$

$$f'(x) = \frac{x(2x) - (x^2 + 9)(1)}{x^2} = \frac{2x^2 - x^2 - 9}{x^2} = \frac{x^2 - 9}{x^2}$$

$$f''(x) = \frac{x^2(2x) - (x^2 - 9)(2x)}{(x^2)^2} = \frac{\cancel{2}x^3 - \cancel{2}x^3 + 18x}{x^4} = \frac{18x}{x^4} = \frac{18}{x^3}$$

$$f'(x) = \frac{x^2 - 9}{x^2}$$

$f'(x)$ is undefined @ $x=0$ NOT a crit. point

$$f'(x) = 0 \rightarrow x^2 - 9 = 0 \rightarrow (x-3)(x+3) = 0$$

$$\rightarrow x = \pm 3$$

$$f(x) = \frac{x^2 + 9}{x}$$

$$f(3) = \frac{9+9}{3} = \frac{18}{3} = 6$$

$$f(-3) = \frac{9+9}{-3} = -6$$

critical points!

$(3, 6), (-3, -6)$

$$f''(x) = \frac{14}{x^3}$$

$f''(x)$ und @ $x=0 \rightarrow$ NOT inf. point

$f''(x) = 0 \rightarrow$ never

no inflection points

crit. points $(3, 6)$ $(-3, -6)$

$$f'(x) = \frac{x^2 - 9}{x^2}$$

$\rightarrow f'(x)$:

A sign chart for the derivative f'(x) = (x^2 - 9)/x^2. The x-axis is marked with vertical lines at x = -3, x = 0, and x = 3. Above the axis, the sign of the derivative is indicated: a '+' sign is above the line to the left of -3, a '-' sign is above the line between -3 and 0, a '-' sign is above the line between 0 and 3, and a '+' sign is above the line to the right of 3.

$$f(-4) = \frac{9}{16}$$

$$f(-1) = -\frac{8}{1}$$

$$f(1) = -\frac{8}{1}$$

$$f(4) = \frac{9}{16}$$

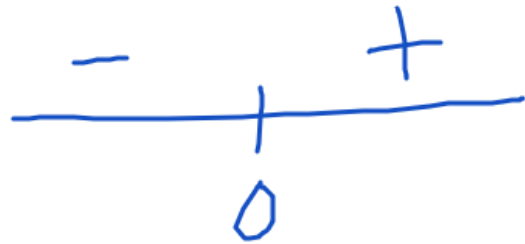
$f(x)$

$\uparrow (-\infty, -3) \cup (3, \infty)$

$\downarrow (-3, 0) \cup (0, 3)$

$$f''(x) = \frac{18}{x^3}$$

$$f''(x) =$$



$$f(-1) = -18$$

$$f(1) = 18$$

$f(x)$
concave up $(0, \infty)$
" down $(-\infty, 0)$

$$f'(x) = \begin{array}{c} + \quad | \quad - \quad | \quad - \quad | \quad + \\ \hline \text{---} -3 \quad \text{---} 0 \quad \text{---} 3 \quad \text{---} \end{array}$$

$$f''(x) = \begin{array}{c} \quad \quad \quad - \quad \quad \quad | \quad \quad \quad + \\ \hline \text{---} \quad \quad \quad 0 \quad \quad \quad \text{---} \end{array}$$

$$f(x) = \begin{array}{c} \begin{array}{c} \nearrow \quad \searrow \quad \searrow \quad \nearrow \\ \text{---} \quad \quad \quad | \quad \quad \quad | \quad \quad \quad \end{array} \\ \hline \text{---} -3 \quad \text{---} 0 \quad \text{---} 3 \quad \text{---} \end{array}$$

