

$$f(x) = \frac{x^2 + 9}{x}$$

- a) find the domain of  $f(x) = (-\infty, 0) \cup (0, \infty) \checkmark$
- b) find any asymptotes VA @  $x=0$ , no HA  $\checkmark$
- ~~c) find  $f'(x)$ ,  $f''(x) \checkmark$~~
- d) identify any critical points  $\checkmark$
- e) " " inflection points  $\checkmark$
- f) find where  $f$  is increasing/decreasing  $\checkmark$
- g) " " " " concave up/down
- h) graph  $f(x)$   $\leftarrow$  "approximately"

$$\lim_{x \rightarrow \infty} \frac{x^2 + 9}{x} \xrightarrow{L'H} \lim_{x \rightarrow \infty} \frac{2x}{1} = 2x \rightarrow \infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 + 9}{x} \xrightarrow{L'H} \lim_{x \rightarrow -\infty} 2x \rightarrow -\infty$$

no H.A.

$$f(x) = \frac{x^2 + 9}{x}$$

$$f'(x) = \frac{x(2x) - (x^2 + 9)}{x^2} = \frac{x^2 - 9}{x^2}$$

$$f''(x) = \frac{x^2(2x) - (x^2 - 9)(2x)}{(x^2)^2} = \frac{18}{x^3}$$

$$f'(x) = \frac{x^2 - 9}{x^2}$$

$f'(x)$  is undefined @  $x=0$  not a crit point

$$f'(x) = 0 \rightarrow x^2 - 9 = 0 \rightarrow (x+3)(x-3) = 0 \rightarrow x = \pm 3$$

$$f(x) = \frac{x^2 + 9}{x}$$

$$f(3) = \frac{9+9}{3} = \frac{18}{3} = 6$$

$$f(-3) = \frac{9+9}{-3} = \frac{18}{-3} = -6$$

crit. points

$(3, 6), (-3, -6)$

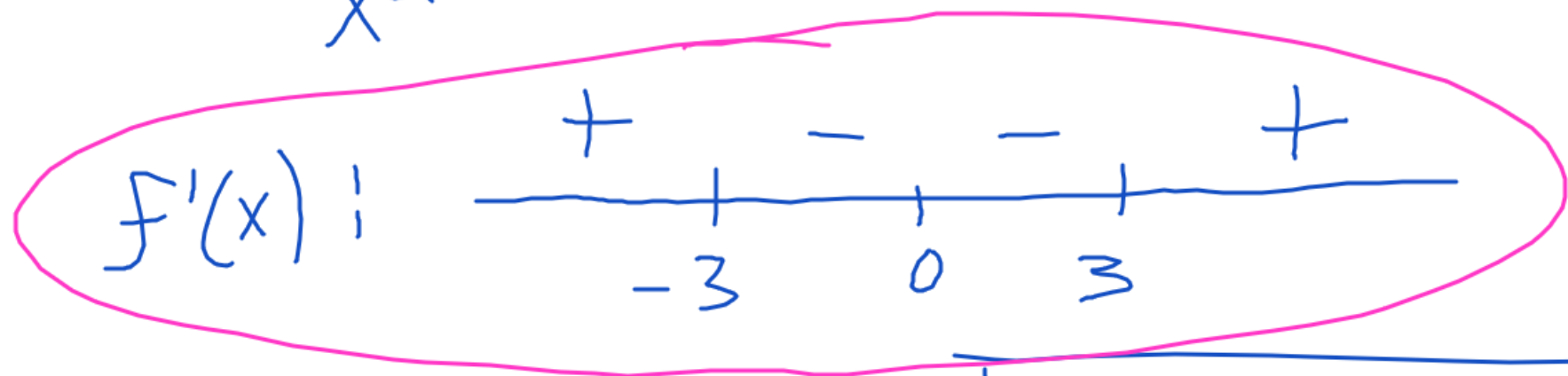
$$f''(x) = \frac{18}{x^3}$$

$f''(x)$  is undefined @  $x=0$  not an inf. pt.

$f''(x)$  is never zero

→ no inflection points

$$f'(x) = \frac{x^2 - 9}{x^2}$$

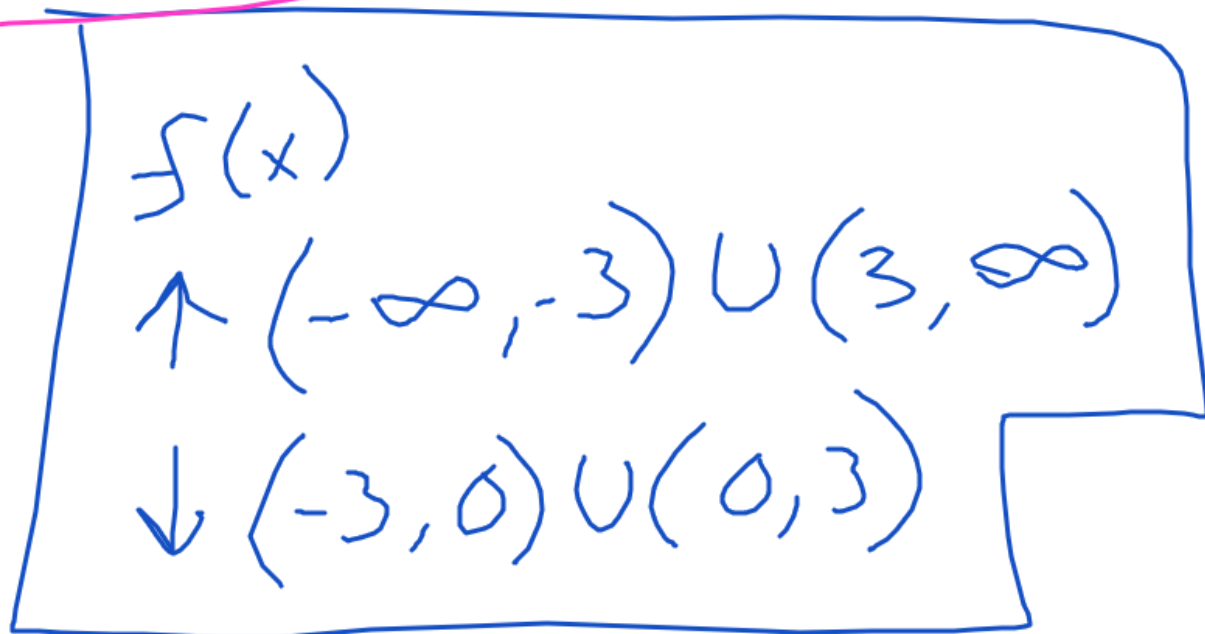


$$f(-4) = \frac{25-9}{16} = \frac{16}{16} = 1$$

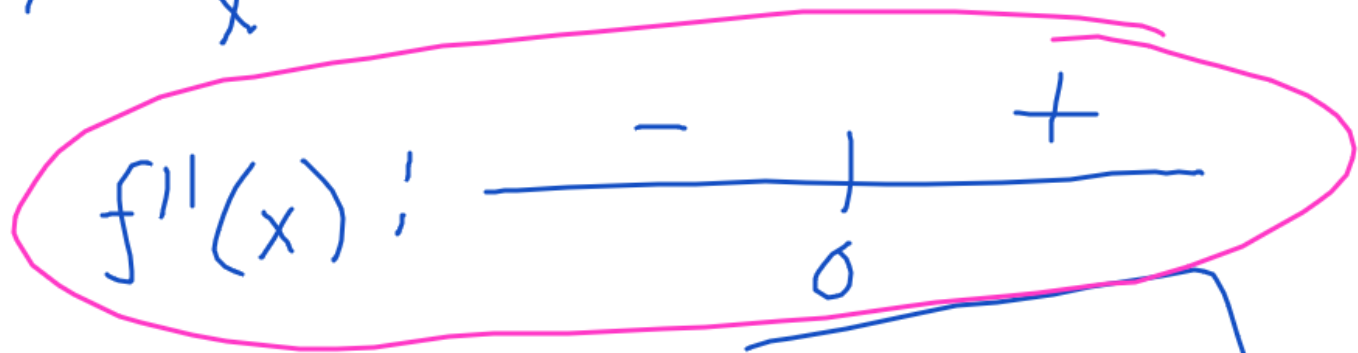
$$f(-1) = \frac{1-9}{1} = -8$$

$$f(1) = \frac{1-9}{1} = -8$$

$$f(4) = \frac{25-9}{16} = 1$$



$$f''(x) = \frac{18}{x^3}$$



$$f''(-1) = -18$$

$$f''(1) = 18$$

$f(x)$   
concave up  $(0, \infty)$   
" down  $(-\infty, 0)$

