FRQ #3

Fall 2020 exam v. A

$$\rightarrow \underbrace{5\sqrt{x}} - \underbrace{2x} - \underbrace{2^x} = 0$$

cont. $[0,\infty)$ $\quad(-\infty,\infty)\quad(-\infty,\infty)$

cont. $[0,\infty)$

$f(0) \rightarrow -2^0 = -1 < 0$ ✗

$f(1) \rightarrow 5-2-2 = 1 > 0$

$\boxed{[0,1]}$, $[1,4]$

IVT

1) contuous ✓

2) $f(a) < 0 < f(b)$

$f(4) = 10-8-16 = -14$ ✗

the function has a solution on $[0,1]$ by the IVT, because ② $\underbrace{f(0)}_{-1} < 0 < \underbrace{f(1)}_{1}$ and $f(x)$ is ① continuous on $[0,\infty)$, so is continuous on $[0,1]$ ■

FRQ #1

$$f(x)=\frac{2e^x+3}{e^x-1}$$

a) vertical asymptotes:

$$e^x-1=0 \rightarrow e^x=1$$

$$\ln(e^x)=\ln(1)$$

$$\boxed{\text{V.A.}\quad x=0}$$

$$f(x) = \frac{2e^x+3}{e^x-1}$$

b) H.A.

$$\lim_{x\to\infty} \frac{2e^x+3}{e^x-1} \to \lim_{x\to\infty} \frac{\frac{2e^x}{e^x}+\frac{3}{e^x}}{\frac{e^x}{e^x}-\frac{1}{e^x}} \to \boxed{\lim_{x\to\infty} \frac{2+\frac{3}{e^x}\}\to 0}{1-\frac{1}{e^x}\}\to 0}} = \frac{2}{1} = 2$$

$$\boxed{y=2,\ y=-3}$$

$$\lim_{x\to-\infty} \frac{2e^x+3}{e^x-1} \to \frac{2e^{-10000}+3}{e^{-10000}-1} \to \frac{\overbrace{\frac{2}{e^{10000}}}^{=0}+3}{\underbrace{\frac{1}{e^{10000}}}_{=0}-1} = \frac{3}{-1} = \boxed{-3}$$

-10000

$$\lim_{x \to -\infty} \frac{2 + \frac{3}{e^x}}{1 - \frac{1}{e^x}}$$

$$\frac{2 + \frac{3}{e^{-10000}}}{1 - \frac{1}{e^{-10000}}} \rightarrow \frac{2 + 3e^{10000}}{1 - e^{10000}} = \frac{\infty}{-\infty}$$

Ind.

$$e^{-2} \rightarrow \frac{1}{e^2}$$

$$\frac{1}{e^{-2}} \rightarrow e^2$$

FRQ #4

$$\lim_{x\to 0} 3x^2 \sin\left(\frac{1}{9x}\right) = 0$$ By the Squeeze Thm.

$$\boxed{0 \le ♥ \le 0}$$

$\rightarrow$

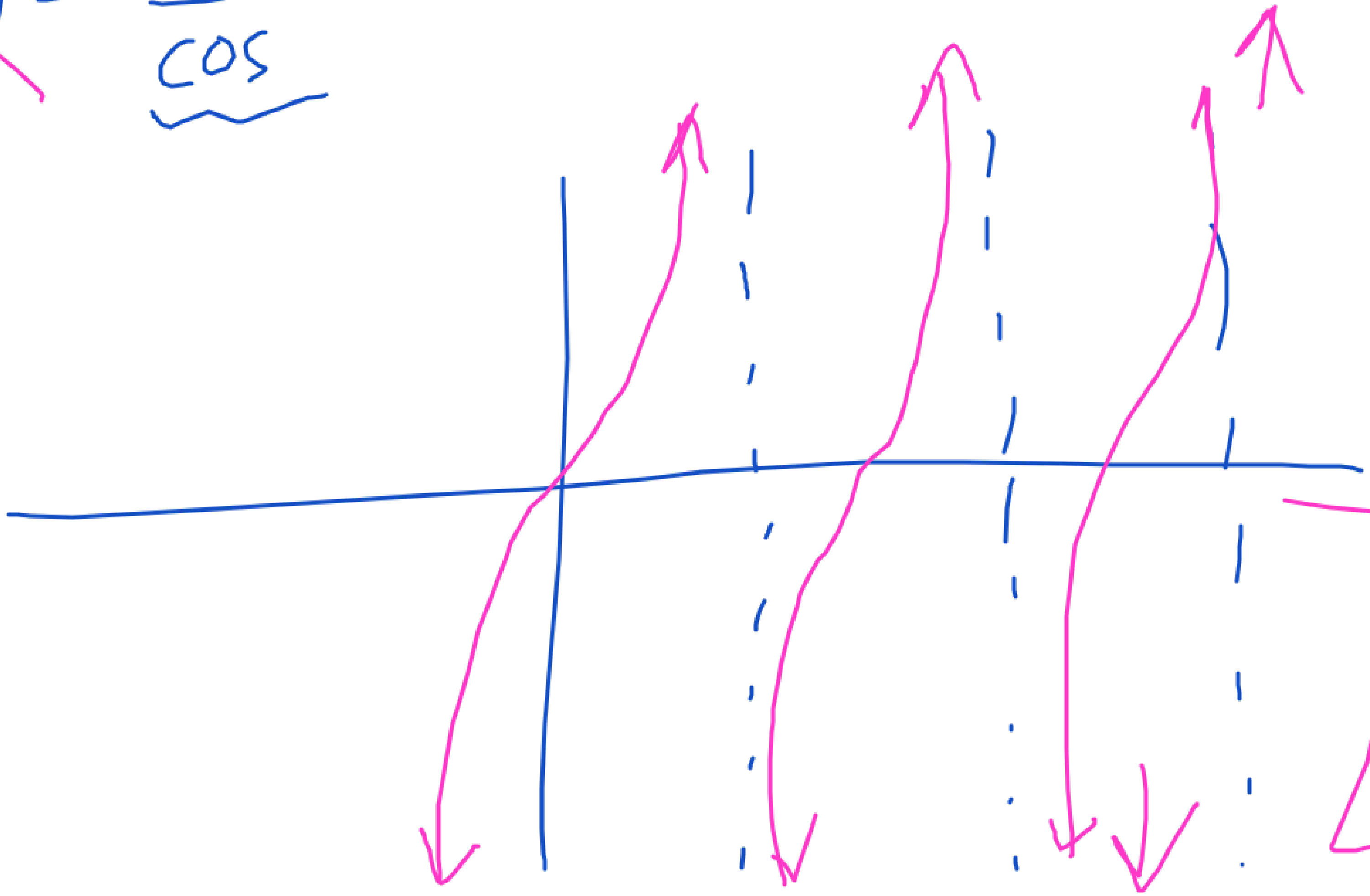
$$-1 \le \sin(x) \le 1$$

$$\rightarrow \left(-1 \le \sin\left(\frac{1}{9x}\right) \le 1\right) 3x^2$$

$$\left(-3x^2 \le 3x^2 \sin\left(\frac{1}{9x}\right) \le 3x^2\right) \lim_{x\to 0}$$

$$\underbrace{\lim_{x\to 0} -3x^2}_{\boxed{=0}} \le \underbrace{\lim_{x\to 0} 3x^2 \sin\left(\frac{1}{9x}\right)}_{♥} \le \underbrace{\lim_{x\to 0} 3x^2}_{\boxed{=0}}$$

$\cancel{\tan(x)} = \dfrac{\sin}{\cos}$



$-1 \le \cos \le 1$

$-1 \le \sin \le 1$

$\boxed{f(2) = 4}$

$f'(x) = 3^x$

tangent line @ $\boxed{x = 2}$

$\boxed{y - y_1 = m(x - x_1)}$

m - slope → $f'(2) = 3^2 = 9$

(x_1, y_1) - point → $(2, 4)$

$y - 4 = 9(x - 2)$

$y - 4 = 9x - 18$

$\boxed{y = 9x - 14}$

FRQ #2

$$f(x) = \sqrt{2x+1} \rightarrow (2x+1)^{1/2} \rightarrow \frac{1}{\cancel{2}}(2x+1)^{-1/2} \cdot \cancel{2}$$

$$= \frac{1}{\sqrt{2x+1}} \checkmark$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$\lim_{h \to 0} \frac{2(x+h)+1 - (2x+1)}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} \rightarrow \lim_{h \to 0} \frac{\cancel{2x} + \boxed{2\cancel{h}} \cancel{+1} - \cancel{2x} - \cancel{1}}{\cancel{h}\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$$

$$\lim_{h \to 0} \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} = \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} = \boxed{\frac{1}{\sqrt{2x+1}}}$$

$f(x) = \frac{g(x)}{h(x)}$

$f(3) = \boxed{\frac{0}{0}}$

A) V.A @ $x = 3$?

B) hole @ $(3, 1)$? ★

$\boxed{g, h \text{ cont.}}$

$g(3) = 0 = h(3)$

$\lim_{x \to 3} f(x) = 1$

DNE

∞

$-\infty$

$\lim_{x \to \infty, -\infty}$