HW 16 #15

$$f(x) = \begin{cases} 6(x-3)^3 - 1, & x < 4 \\ \ln(x-3) - 10, & x \geq 4 \end{cases} \qquad [-4, 14]$$

EVT: $f(x)$ is continuous on $[-4, 14]$

$$\left.\begin{aligned} 6(4-3)^3 - 1 &= 6 - 1 = 5 \\ \underbrace{\ln(4-3)}_{\ln(1) = 0} - 10 &= -10 \end{aligned}\right\} \text{ jump discontinuity}$$

$\rightarrow$ $\boxed{\text{EVT cannot hold}}$

HW 16 #16

$$f(x) = \begin{cases} 4e^{(x-2)} - 1 & x < 2 \\ 5e^{(x-2)} + 8 & x \geq 2 \end{cases}$$

$$\left.\begin{array}{l} 4\underbrace{e^{(2-2)}}_{1} - 1 = 4 - 1 = 3 \\ 5\underbrace{e^{(2-2)}}_{1} + 8 = 5 + 8 = 13 \end{array}\right\} \neq \text{ jump discont,} \rightarrow \boxed{\text{EVT cannot hold}}$$

MVT practice:

$f(x) = x^{2/3}$ on $[-1,1]$

1) continuous on $[-1,1]$ ✓

2) differentiable on $(-1,1)$ ✗

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$ not defined @ $x=0$

MVT does <u>not</u> hold

$$f(x) = \underbrace{e^x}_{(-\infty,\infty)} + \underbrace{\ln(x)}_{(0,\infty)} \qquad \underline{[-1,1]}$$

1) cont. on $[-1,1]$ X

2) diff. on $(-1,1)$

MVT does not hold

$f(x) = x^2\cos(x)$ on $[-\pi,\pi]$

1) cont. on $[-\pi,\pi]$ ✓

2) diff on $(-\pi,\pi)$ ✓

$f'(x) = 2x\cos x - x^2\sin x$ derivative defined on $(-\pi,\pi)$

MVT does hold

Rolle's Thm.

$f(x) = x^2 \cos x \quad [-\pi, \pi]$

1) cont. on $[a,b]$ ✓
2) diff on (a,b) ✓
3) $f(a) = f(b)$ ✓

Rolle's thm. does hold

$$\left.\begin{aligned} f(-\pi) &= (-\pi)^2 \cos(-\pi) = \pi^2(-1) = -\pi^2 \\ f(\pi) &= (\pi)^2 \cos(\pi) = \pi^2(-1) = -\pi^2 \end{aligned}\right\} =$$

If Rolle's thm holds for $f(x)$ on $[a,b]$, then $f(x)$ has <u>at</u> <u>least</u> one horizontal tangent line on $[a,b]$

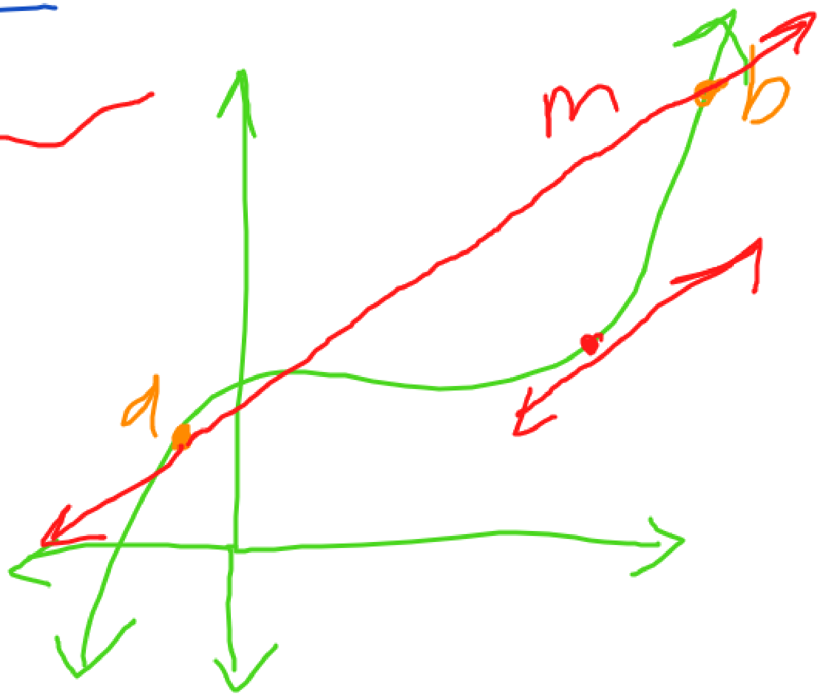
$$\exists c \in [a,b]$$

$$\boxed{f'(c) = 0}$$

MVT)

1) continuous on $[a,b]$
2) differentiable on (a,b)

$\left.\right\}$ $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

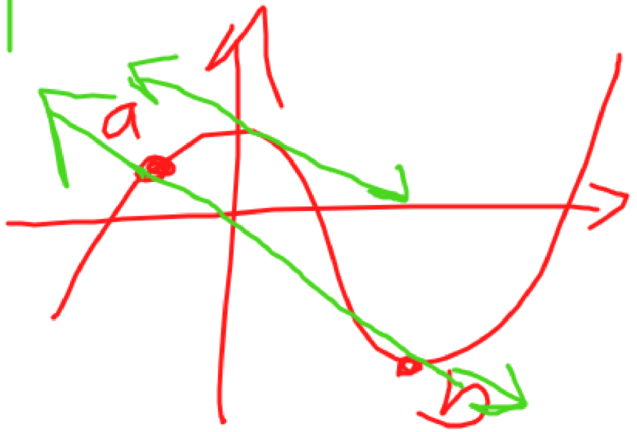


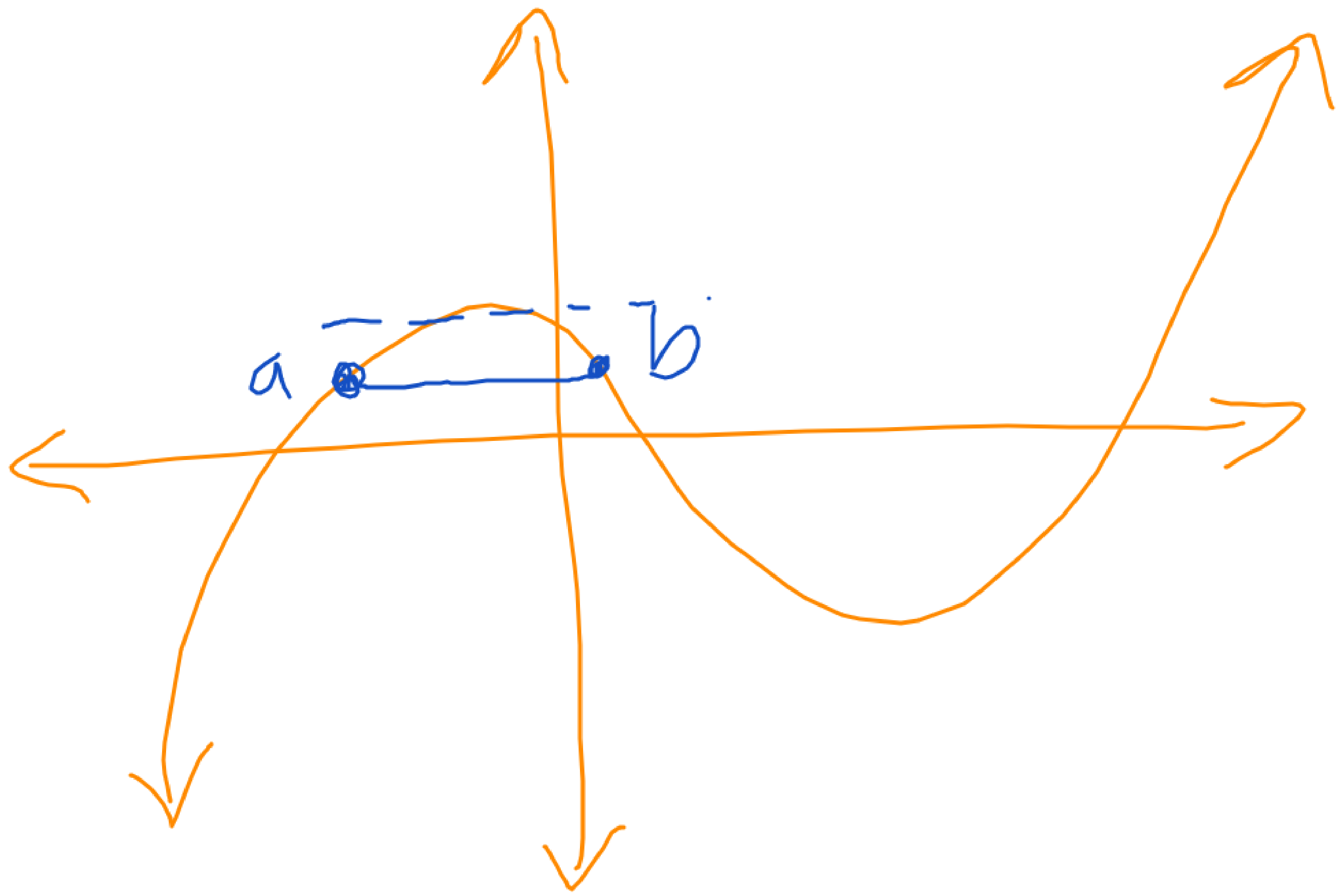
Rolle's)

1) cont. on $[a,b]$
2) diff. on (a,b)
3) $\boxed{f(a)=f(b)}$

$\left.\right\}$ $f'(c)=0$

MVT




a
b

$f(x) = 2x^3 - 9x^2 + 12x + 1$ $[0,3]$

find abs. max/min if they exist

Step 0.5) Extreme value theorem:

$f(x)$ cont. on $[0,3]$ ✓

Step 1) find critical points

$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$

$0 = 6(x-2)(x-1)$

$\rightarrow \boxed{x=2 \quad x=1}$ critical points step 2) find f(x) for critical / end points

end
$f(0) = 1$
$f(3) = 54 - 81 + 36 + 1 = 10$

crit
$f(1) = 2 - 9 + 12 + 1 = 6$
$f(2) = 16 - 36 + 24 + 1 = 5$

step 3) highest = abs. max → $(3, 10)$

lowest = abs. min → $(0, 1)$