$$f(x) = 4\sin^2(x+1) + \cos^2(x+1) \rightarrow [0, 2\pi]$$

$$= 4(\sin(x+1))^2 + [\cos(x+1)]^2$$

$$f'(x) = 8\sin(x+1)\cos(x+1) + 2\cos(x+1)(-\sin(x+1))$$

$$0, \cancel{und} = 6\sin(x+1)\cos(x+1)$$

$$0 = \sin(x+1) \mid \cos(x+1)$$

$$\sin(x+1) = 0$$

$$\underline{\cos(x+1) = 0}$$

$\sin(x) = 0$

$[0, 2\pi]$

$x = 0, \pi, 2\pi, 3\pi, \ldots$

$\sin(x+1) = 0$

$x+1 = 0, \quad x+1 = \pi, \quad x+1 = 2\pi, \quad x+1 = 3\pi \ \ldots$

$\cancel{x = -1}, \quad x = \pi - 1, \quad x = 2\pi - 1, \quad \cancel{x = 3\pi - 1}, \ldots$

$$\boxed{\begin{aligned} x &= \pi - 1 \\ x &= 2\pi - 1 \end{aligned}}$$

$\cos(x) = 0 \qquad [0, 2\pi]$

$\rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\cos(x+1) = 0$

$x+1 = \frac{\pi}{2}, \quad x+1 = \frac{3\pi}{2}, \quad x+1 = \frac{5\pi}{2}, \quad x+1 = \frac{7\pi}{2}, \ldots$

$x = \frac{\pi}{2} - 1, \quad x = \frac{3\pi}{2} - 1, \quad x = \frac{5\pi}{2} - 1, \quad \cancel{x = \frac{7\pi}{2} - 1}$

$\boxed{x = \frac{\pi}{2} - 1, \quad x = \frac{3\pi}{2} - 1, \quad x = \frac{5\pi}{2} - 1}$

HW 19 #6

$$\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right) \qquad x = \frac{1}{1/x}$$

$$\lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{x}\right)}{1/x} \qquad \frac{\sin \to 0}{1/x \to 0}$$

$$\frac{d}{dx}\sin\left(\frac{\pi}{x}\right) \to \pi\cos\left(\frac{\pi}{x}\right)\underbrace{\frac{d}{dx}\frac{1}{x}} = \pi\cos\left(\frac{\pi}{x}\right)\cdot\frac{-1}{x^2}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$$

$$\lim_{x\to\infty} \frac{\pi \cos\left(\frac{\pi}{x}\right) \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$$

$$\lim_{x\to\infty} \pi \cos\left(\frac{\pi}{x}\right)$$

$$\frac{\pi}{x} \to 0$$

$$\pi \cos(0) = \pi \cdot 1 = \boxed{\pi}$$

MVT $f(x) = \ln(x^2)$ $[1, \sqrt{e}]$

- cont $[1, \sqrt{e}]$ ✓

- diff $(1, \sqrt{e})$ ✓

MVT holds ✓

$$f'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

$x \neq 0$

$f(x) = \ln(x^2)$

$[1, \sqrt{e}]$

$\uparrow$ a $\quad \uparrow$ b

Rolle's Thm?

- cont $[1, \sqrt{e}]$ ✓
- diff $(1, \sqrt{e})$ ✓
- $f(a) = f(b)$ ✗

$f(1) = \ln((1)^2) = \ln(1) = 0$

$f(\sqrt{e}) = \ln(\sqrt{e}^2) = \ln(e) = 1$

Rolle's thm fails

$$f(x) = x^4 e^{-x}$$

$$f'(x) = 4x^3 e^{-x} - x^4 e^{-x}$$

$$\frac{0, \cancel{\text{und}} = x^3 (4-x)}{e^x}$$

$$0 = x^3(4-x)$$

→ $\underline{x=0}$, $\underline{x=4}$ critical points

• ↑ ↓ • $\cup$ $\cap$

$-$	$+$	$-$
	0	4

$f'(\underline{-1}) = - \cdot + = -$

$f(1) = + \cdot + = +$

$f(5) = + \cdot - = -$

$$\boxed{\begin{array}{l} f(x) \uparrow \ (0,4) \\ f(x) \downarrow \ (-\infty,0) \cup (4,\infty) \end{array}}$$

$f'(x) = 4x^3e^{-x} - x^4e^{-x}$

$f''(x) = 12x^2e^{-x} - 8x^3e^{-x} + x^4e^{-x}$

$0, \cancel{und} = \frac{x^2(12-8x+x^2)}{e^x}$

$0 = x^2(x^2 - 8x + 12)$

$0 = x^2(x-2)(x-6)$

possible infl. points $\boxed{x=0,\ x=2,\ x=6}$