

HW 22 #10

$$f''(x) = -\cos(x) + \sin(x)$$

$$f'(x) = -\sin(x) - \cos(x) + C$$

$$f(x) = \cos(x) - \sin(x) + \frac{1}{\pi}x$$

$$f(0) = 1 = \cos(0) - \sin(0) + C + D$$

$$1 = 1 + D \rightarrow D = 0$$

$$f(\pi) = 0 = \cos(\pi) - \sin(\pi) + C + D$$

$$0 = -1 + C \rightarrow C = \frac{1}{\pi}$$

✓
 ~~$f(0) = 1, f(\pi) = 0$~~

$$\text{Ex) } \underline{f(x) = 2x^2 - 1} \quad \underline{[2, 5]} \leftarrow$$

Left endpoint approx. with $n=3$

$$\Delta x = \frac{5-2}{3} \rightarrow \Delta x = 1$$

$$\rightarrow [2, 3] [3, 4] [4, 5] \leftarrow$$

$$x = 2, 3, 4$$

$$A = [f(2) + f(3) + f(4)] \Delta x$$

$$= [7 + 17 + 31] (1)$$

$$= 55$$

$$f(x) = 2x^2 - 1 \quad [2, 5]$$

right endpoints $n = 6$

$$\Delta x = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$[2, 2.5] \quad [2.5, 3] \quad [3, 3.5] \quad [3.5, 4] \quad [4, 4.5] \quad [4.5, 5]$$

$$A = \left[f\left(\frac{5}{2}\right) + f(3) + f\left(\frac{7}{2}\right) + f(4) + f\left(\frac{9}{2}\right) + f(5) \right] \left(\frac{1}{2}\right)$$
$$= \left[\frac{25}{2} - 1 + 17 + \frac{49}{2} - 1 + 31 + \frac{81}{2} - 1 + 49 \right] \left(\frac{1}{2}\right)$$

$$A = \left[\frac{155}{2} + 97 - 3 \right] \left(\frac{1}{2} \right)$$

$$\left[\frac{155 + 188}{2} \right] \left(\frac{1}{2} \right)$$

$$= \frac{343}{2} \cdot \frac{1}{2} = \boxed{\frac{343}{4}}$$

$$f(x) = 2x^2 - 1 \quad [2, 5] \quad n = 3$$

mid

$$[2, 3] \quad [3, 4] \quad [4, 5]$$

$$\Delta x = 1$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \frac{5}{2} & \frac{7}{2} & \frac{9}{2} \end{array}$$

$$A = \left[f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right) + f\left(\frac{9}{2}\right) \right] \Delta x$$

$$\left(\frac{25}{2} - 1 + \frac{49}{2} - 1 + \frac{81}{2} - 1 \right) (1)$$
$$\left(\frac{23 + 47 + 79}{2} \right) = \boxed{\frac{149}{2}}$$