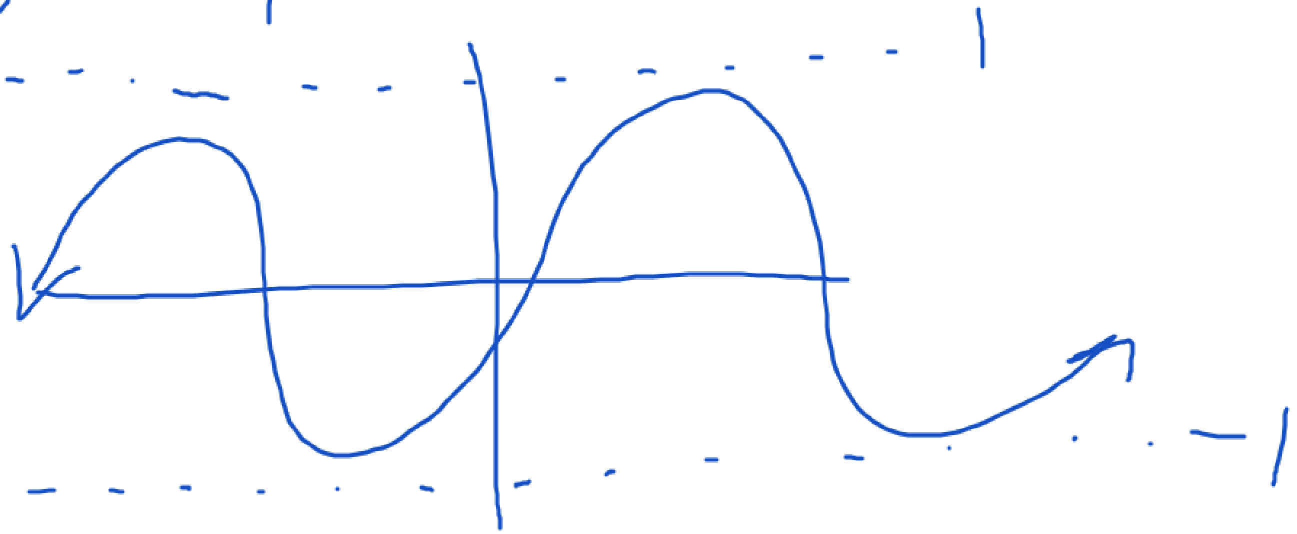
Exam 1 FRQ <u>Musts</u>

1) Write Squeeze Theorem / IVT
2) write $\lim_{x \to \ldots}$ for every necessary step
   - stop writing once the limit is evaluated
3) For IVT, explain why the function is continuous

$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ By the Squeeze Thm.



$\rightarrow \quad -1 \le \sin(x) \le 1$

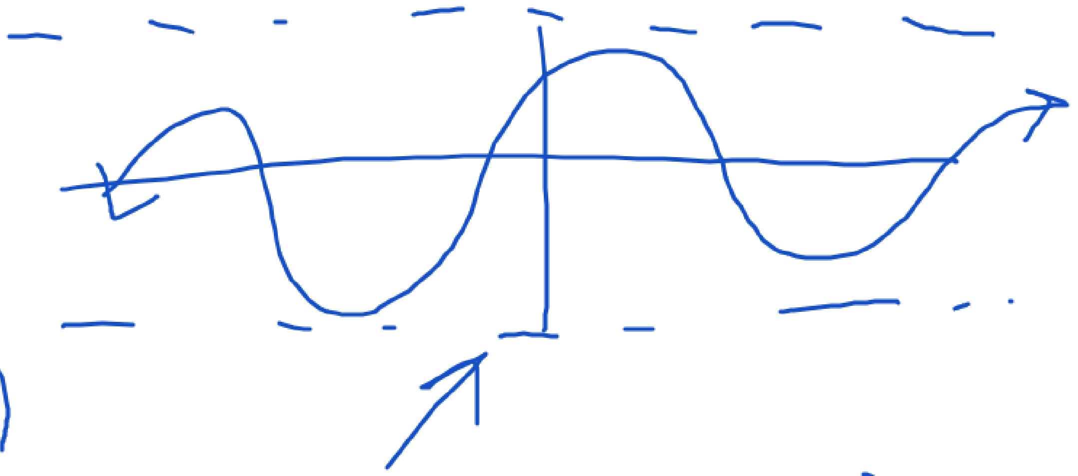
$x^2\left(-1 \le \sin\left(\frac{1}{x}\right) \le 1\right)$

$\lim_{x\to 0}\left(-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2\right)$

$\lim_{x\to 0} -x^2 \le \underbrace{\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)}_{g(x)} \le \lim_{x\to 0} x^2$

$0 \le g(x) \le 0$

$$\lim_{x \to \infty} \frac{2-\cos x}{x+3} = 0$$



$$(-1 \le \cos x \le 1)(-1)$$

$$(1 \ge -\cos x \ge -1)(+2)$$

$$\left(\frac{3}{x+3} \ge \frac{2-\cos x}{x+3} \ge \frac{1}{x+3}\right)\left(\lim_{x \to \infty}\right)$$

$$\lim_{x \to \infty} \frac{3}{x+3} \ge \underbrace{\lim_{x \to \infty} \frac{2-\cos x}{x+3}}_{g(x)} \ge \lim_{x \to \infty} \frac{1}{x+3}$$

$$0 \ge g(x) \ge 0$$

By the squeeze thm.

$$f(x) = \begin{cases} e^{kx} & 0 \le x \le 3 \\ x+3 & x > 3 \end{cases}$$

$$\ln(e^{3k}) = \ln(6)$$

$$3k = \ln(6)$$

$$k = \frac{\ln(6)}{3}$$

$$g(x) = \ln(x) + \frac{e^x}{x^2-1}$$

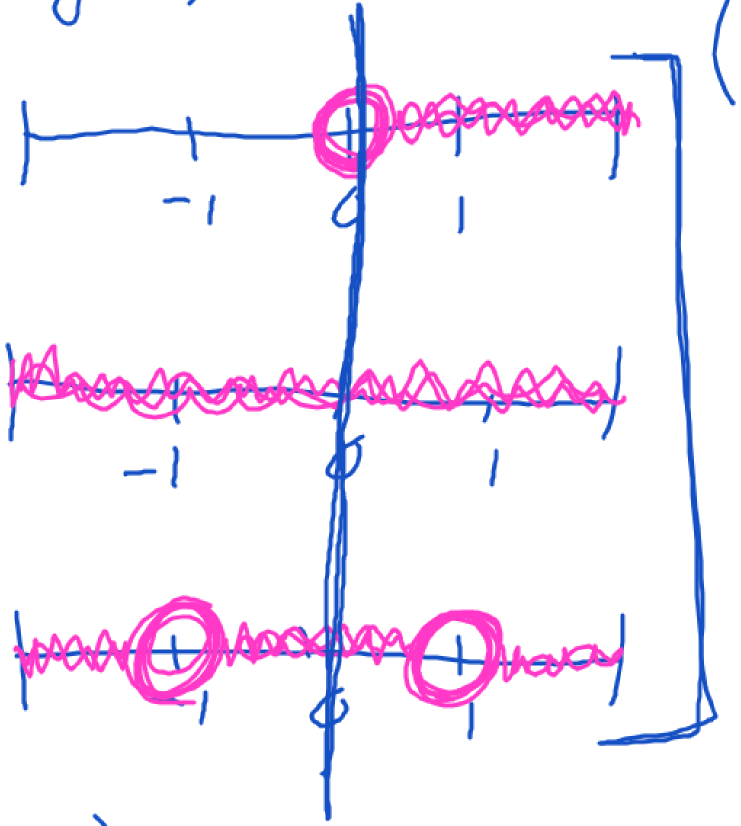
det. which int. $g(x)$ is cont.

1) $\ln(x) \rightarrow (0, \infty)$

2) $e^x \rightarrow (-\infty, \infty)$

3) $\frac{1}{x^2-1} \rightarrow x \neq 1$, $x \neq -1$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$



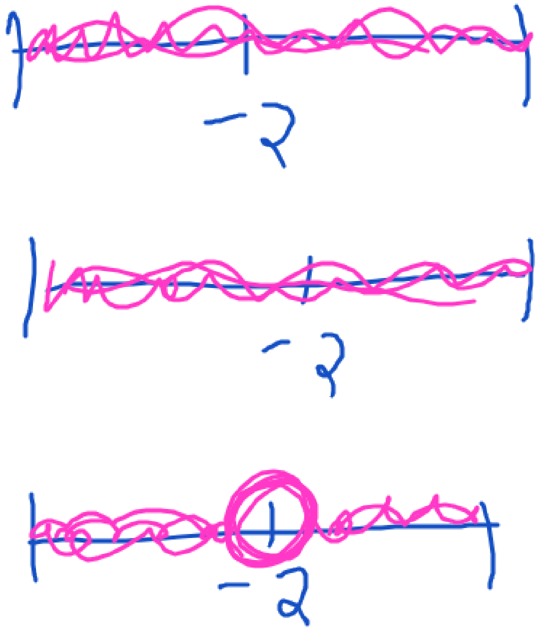
$(0, 1) \cup (1, \infty)$

$$g(x) = \sqrt[3]{x+1} - \frac{\cos(x)}{x+2}$$

1) $\sqrt[3]{x+1} \longrightarrow (-\infty, \infty)$

2) $-\cos(x) \longrightarrow (-\infty, \infty)$

3) $\frac{1}{x+2} \longrightarrow x \neq -2$



$(-\infty, -2) \cup (-2, \infty)$