MAC 2311

December 3, 2020 **Quiz 12: Last one!**

Remember to show <u>all</u> of your work.

Problem 1. Determine the expression for $f(x)$ given the following information:

- $f''(x) = 30x + 32 - e^x$
- $f(0) = 0$
- $f(1) = 25 - e$

Sample Solution: We start by taking the antiderivative twice:

$$\begin{aligned} f''(x) &= 30x + 32 - e^x \\ f'(x) &= 15x^2 + 32x - e^x + C \\ f(x) &= 5x^3 + 16x^2 - e^x + Cx + D \end{aligned}$$

Now we have to use the initial conditions to determine C and D. Beginning with $f(0) = 0$:

$$\begin{aligned} f(x) &= 5x^3 + 16x^2 - e^x + Cx + D \\ f(0) &= 5(0)^3 + 16(0)^2 - e^0 + C(0) + D \\ 0 &= -1 + D \\ D &= 1 \end{aligned}$$

and now with $f(1) = 25 - e$:

$$\begin{aligned} f(x) &= 5x^3 + 16x^2 - e^x + Cx + 1 \\ f(1) &= 5(1)^3 + 16(1)^2 - e^1 + C(1) + 1 \\ 25 - e &= 5 + 16 - e + C + 1 \\ 25 - e &= 22 - e + C \\ 25 &= 22 + C \\ C &= 3 \end{aligned}$$

and thus, $f(x) = 5x^3 + 16x^2 - e^x + 3x + 1$

Problem 2. Find an approximation of the area under the graph of $f(x) = x^2 + 1$ on the interval $[1, 13]$ using

(a) right endpoints with $n = 4$

Sample Solution First, note that the length of each interval will be $\frac{13-1}{4} = \frac{12}{4} = 3$. Therefore, the intervals are $[1, 4]$, $[4, 7]$, $[7, 10]$, and $[10, 13]$, and $\Delta x = 3$. Using right endpoints means we will use the x values $x = 4, 7, 10$, and 13. Thus,

$$\begin{aligned} Area &\approx f(4)\Delta x + f(7)\Delta x + f(10)\Delta x + f(13)\Delta x \\ &\approx (4^2+1)(3) + (7^2+1)(3) + (10^2+1)(3) + (13^2+1)(3) \\ &\approx 17(3) + 50(3) + 101(3) + 170(3) \\ &\approx 51 + 150 + 303 + 510 \\ &\approx 1014 \end{aligned}$$

(b) midpoints with $n = 3$

Sample Solution First, note that the length of each interval will be $\frac{13-1}{3} = \frac{12}{3} = 4$. Therefore, the intervals are $[1, 5]$, $[5, 9]$, and $[9, 13]$, and $\Delta x = 4$. Using midpoints means we will use the x values $x = 3, 7$, and 11. Thus,

$$\begin{aligned} Area &\approx f(3)\Delta x + f(7)\Delta x + f(11)\Delta x \\ &\approx (3^2+1)(4) + (7^2+1)(4) + (11^2+1)(4) \\ &\approx 10(4) + 50(4) + 122(4) \\ &\approx 40 + 200 + 488 \\ &\approx 728 \end{aligned}$$