Remember to show <u>all</u> of your work.

Problem 1. (5 points)

Let $\lim_{x\to a} f(x) = 4$, $\lim_{x\to a} g(x) = -1$, and $\lim_{x\to a} h(x) = 5$. Calculate $\lim_{x\to a}\left[\frac{\big(h(x)\big)^2 - 3f(x)}{g(x)}\right]$.

Solution: This is a problem based entirely around the usage of limit laws. First note that each of our given limits is as x approaches a. We can rewrite the problem as

$$\lim_{x\to a}\left[\frac{\big(h(x)\big)^2 - 3f(x)}{g(x)}\right] = \frac{\Big(\lim_{x\to a} h(x)\Big)^2 - 3\Big(\lim_{x\to a} f(x)\Big)}{\lim_{x\to a} g(x)}$$

and thus plugging in the information we're given yields

$$\lim_{x\to a}\left[\frac{\big(h(x)\big)^2 - 3f(x)}{g(x)}\right] = \frac{(5)^2 - 3(4)}{-1} = \frac{25-12}{-1} = \frac{13}{-1} = -13$$

Problem 2. (5 points)

Evaluate $\lim_{x\to 3}\left[\frac{x^2+5}{x^2-6x+9}\right]$.

Solution: The first thing we can try is direct substitution (i.e. plugging in $x = 3$ to our problem). This gives

$$\frac{(3)^2+5}{(3)^2-6(3)+9} = \frac{9+5}{9-18+9} = \frac{14}{0}$$

now because we have a nonzero number over zero, we're inclined to think the function $f(x) = \frac{x^2+5}{x^2-6x+9}$ has a vertical asymptote at $x = 3$. We can check the signs on either side by setting up a number line with choices on either side of $x = 3$.

Take $x = 2$:

$$\frac{(2)^2 + 5}{(2)^2 - 6(2) + 9} = \frac{4 + 5}{4 - 12 + 9} = \frac{9}{1} = 9$$

Take $x = 4$:

$$\frac{(4)^2 + 5}{(4)^2 - 6(4) + 9} = \frac{16 + 5}{16 - 24 + 9} = \frac{21}{1} = 21$$

Thus on either side of $x = 3$ the function is positive. We have

$$\lim_{x \to 3^-} \frac{x^2 + 5}{x^2 - 6x + 9} = \infty, \quad \lim_{x \to 3^+} \frac{x^2 + 5}{x^2 - 6x + 9} = \infty$$

and thus

$$\lim_{x \to 3} \left[\frac{x^2 + 5}{x^2 - 6x + 9} \right] = \infty$$