MAC 2311

September 17, 2020 **Quiz 2**

Remember to show all of your work.

Problem 1. Use the Squeeze Theorem to evaluate the limit, or explain why the theorem should not be used.

$$\lim_{x \to 0} x^4 \sin\left(\frac{4}{x^2}\right)$$

Sample Answer: Here we see a trig function $(\sin(x))$, so our first inclination is that we can indeed use the theorem.

Note that $\sin(x)$ is bounded: $-1 \leq \sin(x) \leq 1$ for all x. Therefore, we only need to make that center function appear like the function in the main problem. We have

$$-1 \leq \sin\left(\frac{4}{x^2}\right) \leq 1$$

Then multiply each term by x^4:

$$-x^4 \leq x^4 \sin\left(\frac{4}{x^2}\right) \leq x^4$$

Then take the limit of each term as $x \to 0$:

$$\lim_{x \to 0} -x^4 \leq \lim_{x \to 0} x^4 \sin\left(\frac{4}{x^2}\right) \leq \lim_{x \to 0} x^4$$

Now, notice that direct substitution works for the left and right terms, and the center term is the one we want to solve for. Plugging in $x = 0$ to the left and right:

$$0 \leq \lim_{x \to 0} x^4 \sin\left(\frac{4}{x^2}\right) \leq 0$$

Thus, by the Squeeze Theorem,

$$\lim_{x \to 0} x^4 \sin\left(\frac{4}{x^2}\right) = 0$$

Problem 2. Determine the interval(s) on which the following function is continuous. Write your answer in interval notation.

$$g(x) = \sqrt{x^2 - 1} + \frac{\sin(x)}{x+2}$$

Sample Answer Here we follow the approach that Dr. York outlined in class. Let's consider each function individually, determine their intervals of continuity, and then combine using a number line.

First, we know that anything under a square root cannot be negative. Therefore,

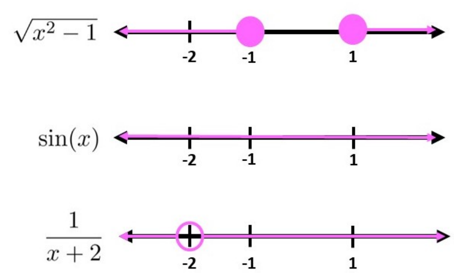
$$\sqrt{x^2 - 1} \geq 0 \longrightarrow x^2 - 1 \geq 0 \longrightarrow x^2 \geq 1 \longrightarrow x \geq 1 \text{ or } x \leq -1$$

Thus, $\sqrt{x^2 - 1}$ is continuous on $(-\infty, -1] \cup [1, \infty)$

Second, the function $\sin(x)$ is continuous everywhere: $(-\infty, \infty)$

Third, the function $\dfrac{1}{x+2}$ is only discontinuous where the denominator is zero, and therefore $x \neq -2$. We have that $\dfrac{1}{x+2}$ is continuous on $(-\infty, -2) \cup (-2, \infty)$

Now we see that the only points we need to worry about are $-2, -1, 1$. Thus we can construct our number lines:



Now wherever we see overlapping pink, that is where the function is continuous.

Thus, $g(x)$ is continuous on $(\infty, -2) \cup (-2, -1] \cup [1, \infty)$.