

Remember to show all of your work.

Problem 1. Find the first and second derivatives of the following function:

$$f(x) = 2x^5e^x + 3x^4 + \pi^2$$

Sample Solution: here we use the product rule and the rule for derivatives of polynomials and constants

$$\begin{aligned} f(x) &= 2x^5e^x + 3x^4 + \pi^2 \\ f'(x) &= \left(\frac{d}{dx}2x^5\right) * e^x + 2x^5 * \left(\frac{d}{dx}e^x\right) + \left(\frac{d}{dx}3x^4\right) + \left(\frac{d}{dx}\pi^2\right) \\ &= (2 * 5)x^{5-1} * e^x + 2x^5 * e^x + (3 * 4)x^{4-1} + 0 \\ &= 10x^4e^x + 2x^5e^x + 12x^3 \end{aligned}$$

Important note: π^2 is a constant, so $\frac{d}{dx}\pi^2 = 0$

$$\begin{aligned} f'(x) &= 10x^4e^x + 2x^5e^x + 12x^3 \\ f''(x) &= \left(\frac{d}{dx}10x^4\right) * e^x + 10x^4 * \left(\frac{d}{dx}e^x\right) + \left(\frac{d}{dx}2x^5\right) * e^x + 2x^5 * \left(\frac{d}{dx}e^x\right) + \left(\frac{d}{dx}12x^3\right) \\ &= (10 * 4)x^{4-1}e^x + 10x^4e^x + (2 * 5)x^{5-1}e^x + 2x^5e^x + (12 * 3)x^{3-1} \\ &= 40x^3e^x + 10x^4e^x + 10x^4e^x + 2x^5e^x + 36x^2 \\ &= 40x^3e^x + 20x^4e^x + 2x^5e^x + 36x^2 \end{aligned}$$

Problem 2. Find the x value(s) at which the following function has a horizontal tangent line:

$$f(x) = x^3 + 10x^2 + 12x + 7$$

Sample Solution: To find horizontal tangent lines, we need to find exactly where the derivative of the function is equal to zero.

$$f(x) = x^3 + 10x^2 + 12x + 7$$

$$f'(x) = (3 * 1)x^{3-1} + (10 * 2)x^{2-1} + (12 * 1)x^{1-1} + 0$$

$$= 3x^2 + 20x + 12$$

and now we need to find where this is zero:

$$0 = 3x^2 + 20x + 12$$

$$= (3x + 2)(x + 6)$$

and therefore the function has horizontal tangent lines at $x = -\frac{2}{3}$ and $x = -6$