MAC 2311

October 8, 2020 **Quiz 5**

Remember to show all of your work.

Problem 1. Calculate the derivative of each function. No need to simplify your answer.

(a) (2 points) $f(x) = e^{\sqrt{3x+1}}$

Sample Solution

$$\begin{aligned} f(x) &= e^{\left((3x+1)^{1/2}\right)} \\ f'(x) &= e^{\left((3x+1)^{1/2}\right)} * \frac{d}{dx}\left((3x+1)^{1/2}\right) \\ &= e^{\left((3x+1)^{1/2}\right)} * \frac{1}{2}(3x+1)^{-1/2} * \frac{d}{dx}(3x+1) \\ &= e^{\left((3x+1)^{1/2}\right)} * \frac{1}{2}(3x+1)^{-1/2} * 3 \end{aligned}$$

(b (2 points) $f(x) = \sec^2(4x+1)$

Sample Solution

$$\begin{aligned} f(x) &= \left(\sec(4x+1)\right)^2 \\ f'(x) &= 2(\sec(4x+1))^1 * \frac{d}{dx}\left(\sec(4x+1)\right) \\ &= 2(\sec(4x+1)) * \sec(4x+1)\tan(4x+1) * \frac{d}{dx}(4x+1) \\ &= 2(\sec(4x+1)) * \sec(4x+1)\tan(4x+1) * 4 \end{aligned}$$

(c) (2 points) $f(x) = \tan\left(e^{x^2}\right)$

Sample Solution

$$\begin{aligned} f'(x) &= \sec^2(e^{x^2}) * \frac{d}{dx}(e^{x^2}) \\ &= \sec^2(e^{x^2}) * e^{x^2} * \frac{d}{dx}(x^2) \\ &= \sec^2(e^{x^2}) * e^{x^2} * 2x \end{aligned}$$

Problem 2. (4 points) Find the equation of the tangent line of the following function at $x = \pi$:

$$f(x) = \frac{e^{\sin(x)}}{\cos(x)} + 3x$$

Sample Solution First, we should find the (x, y) coordinates of the point we want to find the tangent line of. We have $x = \pi$, so it remains to determine $f(\pi)$:

$$\begin{aligned} f(\pi) &= \frac{e^{\sin(\pi)}}{\cos(\pi)} + 3(\pi) \\ &= \frac{e^0}{-1} + 3\pi \\ &= -1 + 3\pi \end{aligned}$$

So the point is $(\pi, -1 + 3\pi)$. Now we have to find the slope. There are a few ways to do this:

First way: quotient + chain rule

$$\begin{aligned} f'(x) &= \frac{\frac{d}{dx}(e^{\sin(x)}) * \cos(x) - e^{\sin(x)} * \frac{d}{dx}(\cos(x))}{\cos^2(x)} + 3 \\ &= \frac{e^{\sin(x)} * \frac{d}{dx}(\sin(x)) * \cos(x) - e^{\sin(x)} * (-\sin(x))}{\cos^2(x)} + 3 \\ &= \frac{e^{\sin(x)} * \cos(x) * \cos(x) - e^{\sin(x)} * (-\sin(x))}{\cos^2(x)} + 3 \\ f'(\pi) &= \frac{e^0(-1)(-1) - e^0(0)}{(-1)^2} + 3 = \frac{1-0}{1} + 3 = 4 \end{aligned}$$

Second way: product + chain rule

$$\begin{aligned} f(x) &= e^{\sin(x)} \sec(x) + 3x \\ f'(x) &= \frac{d}{dx}(e^{\sin(x)}) * \sec(x) + e^{\sin(x)} * \frac{d}{dx}(\sec(x)) + 3 \\ &= e^{\sin(x)} * \frac{d}{dx}(\sin(x)) * \sec(x) + e^{\sin(x)} * \sec(x)\tan(x) + 3 \\ &= e^{\sin(x)} * \frac{d}{dx}(\sin(x)) * \sec(x) + e^{\sin(x)} * \sec(x)\tan(x) + 3 \\ &= e^{\sin(x)} * \cos(x) * \sec(x) + e^{\sin(x)} * \sec(x)\tan(x) + 3 \\ f'(\pi) &= e^0(-1)(-1) + e^0(-1)(0) + 3 = 1 + 0 + 3 = 4 \end{aligned}$$

Therefore, (any of the following are correct, depending on your application of algebra)

$$\begin{aligned} y - (-1 + 3\pi) &= 4(x - \pi) \\ y + 1 - 3\pi &= 4x - 4\pi \\ y &= 4x - \pi - 1 \\ y &= 4x - (\pi + 1) \end{aligned}$$